# PROBABILITY DENSITY OF STRONG AND SATURATED INTENSITY FLUCTUATIONS IN LASER DETECTION AND RANGING OF CORNER-CUBE REFLECTORS THROUGH THE TURBULENT ATMOSPHERE 

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#### Abstract

The experimental data are analyzed on the variance and probability density of the intensity fluctuations when a plane wave is reflected from an array of corner-cube reflectors for $\beta_{0} \sim 2-12$. It is shown that the intensity fluctuations saturate faster and their level decreases as the number of reflectors in the array increases. The transformation of the law of the intensity fluctuations distribution with the increase of the number of corner-cube reflectors in the array and the change of the conditions of propagation has been revealed.


Different statistical characteristics of the intensity fluctuations of an optical wave reflected from a corner-cube reflector were investigated theoretically and experimentally in a number of papers (see, for example, Refs. $1-3$ and the references therein). A case of reflection from a spatial array of corner-cube reflectors used, for example, in systems of measuring the angular coordinates of different objects, is of great practical interest. The intensity fluctuation characteristics were studied in Ref. 4 for the joint effect of the atmospheric turbulence and of the interference of waves reflected from individual reflectors of the array; however, the obtained expression for the probability density of the intensity fluctuations gives the magnitude of the fluctuation variance that does not agree with the experiment. ${ }^{5}$

In this paper, we analyze the behavior of the standard deviation and the probability density of the intensity fluctuations of a plane wave reflected from a two-dimensional matrix of corner-cube reflectors vs. the level of the atmospheric turbulence and the number of corner-cubes in the matrix for strong and saturated intensity fluctuations. The data obtained when a plane wave was reflected from a specular disk on a V-shaped path under the same conditions of propagation are presented for comparison. The model probability densities are compared with the experimental data.

Our measurements were carried out on a horizontal path above an even underlying surface in the daytime in July and August 1995. A quasiplanar wave was formed at the exit from a radiation source by a lens objective 500 mm in diameter with the effective output beam size of approximately 8.5 cm at a level of $e^{-1}$. A reflector was placed at the distance $L=1240 \mathrm{~m}$ from the source. The reflector was a two-dimensional array of high-quality corner-cube reflectors. The diameter of the individual reflector was $\approx 2.6 \mathrm{~cm}$. A diagonal mirror, based on a bevel end
of a quartz cylinder 1 mm in diameter suspended by a thin ( $\approx 0.3 \mathrm{~mm}$ ) wire stretcher in the beam center, was used for selection of a reflected signal. The laser radiation ( $\lambda=0.63 \mu \mathrm{~m}$ ) was received by the FEU-79 photodetector with the input aperture 0.3 mm in diameter.

The measurements were carried out with 1, 4, 16 , and 26 reflectors in the matrix. Measurements with the flat mirror were carried out simultaneously by the technique described in Ref. 6. The turbulent state of the atmosphere was monitored from the intensity fluctuations on a reference path of the length $L_{1}=200 \mathrm{~m}$. Signals from FEU were recorded by a 4 -channel digital tape-recorder with a sampling rate of 5 kHz in each channel during 5 min . High reliability of recording and playing of the digital data (coefficient of skipping was no greater than $10^{-7}$ ) and the dynamic range ( 12 bits) made it possible to estimate reliably histograms in the range of signal variation $0.01 \leq I /\langle I\rangle \leq 20-25$.

The normalized rms values of the intensity fluctuations $\beta=\sqrt{\left\langle I^{2}\right\rangle /\langle I\rangle^{2}-1}$ are shown in Fig. 1 as functions of the parameter $\beta_{0}=1.23 C_{n}^{2} k^{7 / 6} L^{11 / 6}$ (where $C_{n}^{2}$ is the structure characteristic of the refractive index, $k=2 \pi / \lambda$ is the wave number, and $L$ is the path length) characterizing the intensity of the atmospheric turbulence. The curve corresponds to the average experimental data obtained in Ref. 7 for a plane wave on a straight path.

As seen from the data shown in Fig. 1, the fluctuations saturate faster and their level decreases as the number of reflectors in the array of corner-cube reflectors increases. The saturation has not yet been reached up to $\beta_{0} \approx 10$ in the case of one corner-cube. The scintillation index was maximum at $\beta_{0}=9.8$ and was $\beta=2.4$. Saturation also has not yet been observed up to $\beta_{0} \approx 12$ for 4 corners, but the scintillation index
in the focus was less approximately by unity than in the previous case. Saturation can be assumed to be reached at $\beta_{0} \sim 6-8$ for 16 corner-cubes, and its level is $\beta \approx 1.3$. In the case of 26 corner-cubes, fluctuations saturate at the level $\beta=1$ corresponding to the level of saturation on the straight path.


The probability density of the instantaneous amplitudes of the process gives more complete information on the nature of its fluctuations. The histograms of the instantaneous values of intensity were analyzed for 76 realizations and compared with the model distribution functions. The characteristic histograms of the normalized values of intensity are shown in Fig. 2 for 1 and 4 corner-cubes at different $\beta_{0}$. The probability density functions of $K$ distribution
$\langle I\rangle P(I)=(2 / \Gamma(y)) y^{(y+1) / 2} I^{(y-1) / 2} K_{y-1}\left[2(I y)^{1 / 2}\right]$,
$y=2 /\left(\beta^{2}-1\right), y>0$,
are also shown here for comparison, where $K_{\mathrm{v}}(z)$ is the McDonald function and $\Gamma(z)$ is the gamma function. As is seen, $K$-distribution describes well the experimental data for one corner-cube and all considered levels of turbulence (in this case, in the region close to the focus of fluctuations).

The model (1) also fits well the histogram (Fig. 2b) when the wave reflects from four corner-cubes for $\beta_{0}=11$. Slight deviation from $K$-distribution is observed in the deep fading range (Fig. 2a) as $\beta_{0}$ decreases.

Still less probability of fading is seen when the wave reflects from the matrix of 16 or 26 cornercubes (Fig. 3). The experimental values are between the $K$-distribution and lognormal distribution
$P(I)=(\sqrt{2 \pi} \sigma I)^{-1} \exp \left[-\left(1 / 2 \sigma^{2}\right)(\ln I-\xi)^{2}\right]$,
$\sigma^{2}=\ln \left(1+\beta^{2}\right) \xi=\ln \left[\langle I\rangle /\left(1+\beta^{2}\right)^{1 / 2}\right]$.
They are close for $I>\langle I\rangle$ and the differences between them are within the limits of statistical experimental error.


FIG. 2. Comparison of the histograms of normalized values of intensity with $K$-distribution for the plane wave reflected from the corner-cube reflector (1) and from the matrix of four corner-cubes (2) at $\beta_{0} \sim 2$ (a) and $\sim 11$ (b).

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<I>P(I)
$$




FIG. 3. Comparison of the histograms of normalized values of intensity with $K$-distribution and lognormal distribution for the plane wave reflected from the matrix of 16 (1) or 26 (2) corner-cubes as well as from the flat mirror (3 and 4) at $\beta_{0} \sim 3$ (a) and $\sim 7$ (b).

Histograms obtained for the straight propagation have the same behavior as for 16 and 26 corner-cubes in all considered range of variation of the parameter $\beta_{0}$, when the values of the scintillation indices $\beta$ are close to each other (Fig. 3). The probability density approaches $K$-distribution as $\beta$ increases at fixed $\beta_{0}$. It is especially well seen in the regime of strong focusing, when the level of fluctuations of the plane wave,
reflected from the matrix of 16 or 26 corner-cubes, is essentially lower than that on the straight path.

The results obtained can be explained as follows. When the plane wave is reflected from the matrix of corner-cubes, it is divided into the narrow collimated beams. Random wanderings of such a beam as a whole in the case of reflection from one corner-cube lead to the increase of probability of high spikes and deep fading (which is characteristic of $K$-distribution, in contrast to the lognormal one), which noticeably increase the relative variance of the intensity fluctuations. The size of a shot in the detection plane increases as the number of corner-cubes in the matrix increases, the contribution of random wanderings to the variance decreases, and the fluctuations approach their mean level, which determines the transformation of the distribution law.

In the case of reflection of the spherical wave from the matrix of corner cubes, ${ }^{5}$ each beam reflected from a corner is incident on the source, so the matrix of corner-cube reflectors can be considered the selffocusing system whose focal length coincides with the path length. In contrast to the plane wave, random wanderings of beams reflected from corner-cubes do not make significant contribution to the intensity fluctuations, are determined by variations of an interference pattern in the plane of observation, ${ }^{5}$ and we have the $K$-distribution under conditions of saturation for the case of a large number of corners (12). In this case, the fluctuations saturate at the level corresponding to the focused beam.

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