COOPERATIVE EFFECTS IN A LIGHT-INDUCED DISCHARGE IN AIR

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Received January 22, 1996

We propose, in this paper, a theoretical model of a gas discharge induced by laser radiation. The processes in such a system are self-consistent and strongly depend on boundary conditions. This system possesses a point of bifurcation R_c, $R_c = \nu / k_c^2 D$, where ν is the power injection rate, D is the thermal diffusion coefficient, k_c is the system wave number. At $R > R_c$ various self-organization phenomena (for example, vortices) occur in the discharge plasma. As a result, the formation of a spatiotemporal autowave with a functional structure (induced by an acoustic wave) at a macroscopic level is possible.

1. The light-induced discharge in the Earth's atmosphere, sustained by the neodymium laser radiation, has been the subject of many investigations and used for scientific and technical purposes.[1,2] The discharge of this type was first obtained by Bunkin et al.[3] Further studies have shown that the power needed for sustaining the discharge is $P_c \approx 1$ MW and practically it does not depend on the beam radius. If the power exceeds the critical P_c, the discharge front moves along a light channel at a rate $v_0 \sim 10$ m/s. In this case the front profile remains constant and the value of the rate slowly increases according the law: $v_0 \sim \sqrt{P/P_c - 1}$. The discharge plasma is optically thin (the absorption coefficient $\mu \sim 10^{-2}$ cm^{-1}), and its parameters are on the average constant in time and homogeneous (within the limits of a light beam) in space. The average density of electrons (ions) in plasma is about $2 \cdot 10^{17}$ cm^{-3}, the average temperature is about 1 eV, the state of plasma is close to local thermodynamic equilibrium (LTE).

In Refs. 4 and 5 we have studied the discharge characteristics due to its nonequilibrium state. The measurements carried out enabled us to determine the character of macroscopic fluctuations; we have also discovered that the discharge is the source of intense acoustic waves with the frequency of the order of 10 kHz. The first theoretical model of the discharge was proposed by Raizer.[1] Within the limits of this model, described by a one-dimensional nonlinear equation of thermal conductivity, he has obtained a correct dependence of the front travel rate on the radiation intensity.

In the present paper when constructing the model we proceed from the gas dynamics equations where nonhydrodynamic mechanisms of energy transfer, namely, thermal conductivity and radiation are taken into account. Neglecting the light beam divergence and taking into account the plasma optical transparency, the discharge channel can be considered to have cylindrical symmetry with the characteristic transverse radius r_0.

2. Consider the plasma cylindrical flux with the density ρ, described by hydrodynamics equations for the radial v_r and longitudinal v_z components of the rate $\mathbf{v}$. In the corresponding coordinates, assuming that the perturbed values follow the law $\exp(i\omega t - ikz)$, where ω is the real frequency, k is the wave number, z is the longitudinal coordinate, the equations of motion and continuity take the form:

$$\begin{aligned} &i\rho\,(\omega - k\,v_0) = -\,\partial p'/\partial r; \\ &\rho\,(\omega - k\,v_0) = -\,kp'; \qquad (1) \\ &ikr\,v_r = \partial(r\,v_r)/\partial r. \end{aligned}$$

Here v_0 is the constant velocity of motion in z-direction; p' is the pressure in the wave. From Eqs. (1) it follows that

$$p' = C\,I_0(kr)\,\exp(i\omega t - ikz), \qquad (2)$$

where C is the constant; $I_0(kr)$ is the modified Bessel function. Equations (1) are to be supplemented with the corresponding boundary conditions. The first one follows from the fact that the time derivatives of coordinates of the surface points are to be equal to the velocities of these points. In this case the rate of the discharge front travel is constant, being equal to v_0 on the discharge axis. Hence, we have

$$v_r = i\,(\omega - k\,v_0)r'; \quad v_z = v_0\big|\,z = z_0, \quad r = 0. \qquad (3)$$

Here $z_0 = v_0 t$ is the front coordinate. Under the effect of perturbations, the discharge boundary will be curved that results in a supplementary pressure due to the surface tension.[6] The equation for variable pressure at the boundary $p' = \alpha(k^2 - r^{-2})r'$, where α is the coefficient of the surface tension, is the second boundary condition for the set of equations (1). Using

0235-6880/96/08 686-03 $02.00

these conditions and the first of Eqs. (1), we have $p' = (\alpha/\rho r^2)(k^2 r^2 - 1/\omega - k v_0)\partial p'/\partial r$. Substituting the solution (2) to the above expression we arrive at the dispersion relationship:

$$(\omega - k v_0)^2 = (\alpha/\rho r^3)\, kr\, (k^2 r^2 - 1)\, I_1(kr)/I_0(kr). \quad (4)$$

Perturbations in plasma move together with the discharge, therefore, assuming $k = k_0 + iq$, $k_0 = \omega/v_0$, $q/k_0 \ll 1$, from Eq. (4) we find the condition of convective instability, described by the function

$$q = \frac{1}{r_0 v_0}\left[\frac{\alpha}{\rho r_0}\, kr\, (1 - k^2 r^2)\, \frac{I_1(kr)}{I_0(kr)}\right]^{1/2}. \quad (5)$$

Then by applying Eqs. (3) and (5) to equations (1) we determine the constant $C = \rho q v_0/ik$ in solving Eq. (2). This value of the constant enables us to write the final expressions for hydrodynamic fields in the discharge plasma

$$v_z = v_0\, I_0(kr) \cos(\omega t - kz);$$
$$v_r = v_0\, I_1(kr) \sin(\omega t - kz); \quad (6)$$
$$p' = (q/k)\, \rho\, v_0^2\, I_0(kr) \sin(\omega t - kz).$$

3. Now we show that in the open dissipative system, which is the case with the plasma discharge sustained by laser radiation, the processes of self-organization are possible, which determine the fine space–time structure of waves in plasma and, finally, the character of the discharge motion and sound emission. For this purpose the equation is introduced, describing the variation of entropy s in the system (in a unit volume V).

$$\rho\, T(\mathrm{d}s/\mathrm{d}t) = \mathrm{div}(\varkappa\, \nabla T) + \mu w - \Phi, \quad (7)$$

where w is the laser radiation intensity; $\varkappa$ is the coefficient of thermal conductivity; Φ is the power loss due to self-radiation and heat diffusion in radial direction. From Eq. (7) it follows that the standard state of the system (close to LTE physically) is described by the nonlinear equation

$$F(T) = \mu(T)\, w - \Phi(T) = 0\,|\, T = T_0. \quad (8)$$

Deviation of temperature T' in a wave from its standard value T_0 will give rise to the variation of entropy s of the system.[6] Since these deviations are small, then, varying Eq. (7) by changing T' in the vicinity of T_0 $(\partial F/\partial T\ (T = T_0) = B\mu w/T_0)$ we obtain

$$\frac{\partial}{\partial t}\int s\, \mathrm{d}V =$$

$$= \int\left[\frac{\varkappa}{T_0^2}(\nabla T')^2 + B\,\frac{\mu\, w}{T_0^2}\, T' - B\,\frac{\mu\, w}{T_0^3}\, T'^2\right]\mathrm{d}V. \quad (9)$$

We are interested in the values average in time and over the discharge cross section. Using in Eq. (9) the last of Eqs. (6) and the relation $p' = (\rho/\gamma)\, c_s^2\, (T'/T_0)$, where c_s is the sound velocity in plasma and γ is the constant of adiabatic curve, we obtain

$$\dot{s} = \frac{1}{2}\gamma^2\left(\frac{q}{k}\right)^2\left(\frac{v_0}{c_s}\right)^4 k^2\, \varkappa\left(1 - B\,\frac{\mu\, w}{k^2\, \varkappa\, T_0}\,\frac{\langle I_0^2\rangle}{\langle I_0^2\rangle + \langle I_1^2\rangle}\right). \quad (10)$$

Here $\langle I_m^2\rangle = (1/\pi\, r_0^2)\int_0^{r_0} \pi\, r\, \mathrm{d}r\, I_m^2(k\, r)$ is the average, over the cross section, value of Bessel function. The equation, determining the variation of mechanical energy in the system, is of the form $\dot{\varepsilon} = -T_0\, \dot{s}$. Hence, after substituting Eq. (10) it follows that at the final value of the field w, exceeding certain threshold value w_c,

$$\mu\, w_c = B^{-1}\, k^2\, \varkappa\, T_0\, (1 + \langle I_1^2\rangle/\langle I_0^2\rangle) \quad (11)$$

and the system entropy decreases and the wave energy in plasma gradually increases. We calculate the coefficient of absorption (amplification) of the wave propagating along the axis z. Variation of energy occurs following the law $\exp(-2\beta z)$, where β is determined as $\beta = \dot{\varepsilon}/2v_0\, \varepsilon_0$, $\varepsilon_0 = (1/2)\rho v_0^2$ is the wave total energy (in a unit volume). Using these expressions we can write the following equation:

$$\beta = \frac{k^2\, \varkappa\, T_0\, \gamma^2\, q^2\, v_0^4}{2\rho\, v_0^3\, k^2\, c_s^4}\,(1 - R); \quad R = \frac{\mu\, w}{\mu\, w_c}. \quad (12)$$

Let us calculate the velocity of the discharge front travel specified by the processes of self-organization. For this purpose we write the equation of the front motion in the form:

$$v_0\,\frac{\mathrm{d}T}{\mathrm{d}z} = D\,\frac{\mathrm{d}^2T}{\mathrm{d}z^2} + F; \quad D = \frac{\varkappa}{\rho\, c_p}, \quad F = \frac{\mu\, w - \Phi}{\rho\, c_p}, \quad (13)$$

where D is the heat diffusion coefficient, c_p is the heat capacity at constant pressure, F is the function of energy contribution to the discharge.

Equations of type (13) with the nonlinear function $F(T)$ are well known in the theory of autowaves[7] and together with the boundary condition $\partial v/\partial z = 0\,|_{z = z_0}$ they describe the asymptotically stable solutions in the form of a running front. In our case, using Eq. (11), from Eq. (13) the equation follows:

$$v_0 = 2\, k_0\, D\sqrt{R - 1}\,. \quad (14)$$

From Eqs. (11), (12), and (14) we conclude that the point $R_c = 1$ is the point of bifurcation in the system evolution. At $R > 1$ the system manifests itself as a continuous amplifier and correctly reflects the qualitative dependence of the front velocity on the radiation flux density.

4. Thus, in the system, at a certain power contribution the break of symmetry of the originally homogeneous state occurs, which gives rise to formation, in the discharge, of such a coherent structure as the entropy-vortex wave [from Eq. (6) it follows that $\mathrm{rot}\,\mathbf{v} = 2\mathbf{e}_\varphi\, k\, v_0\, I_1(kr)\cos(\omega t - kz)$], propagating with the discharge velocity. Oscillations of the plasma channel surface, induced by this wave, cause periodic compression and rarefaction of air and thus initiate an acoustic wave. Since the oscillation amplitude is small as compared with the wavelength, the sound propagation is governed by the wave equation for cylindrical acoustic waves.[6] Let us estimate the average energy flow in an acoustic wave $I = \rho_0 c_0 \langle u^2\rangle$, where c_0 is the sound velocity in air, u is the gas speed, ρ_0 is the gas density. Using the condition of mass conservation $\rho_0\, u = \rho\, v_r \big|\, r = r_0$ at the discharge surface and the formula for the radial velocity (6), we write the expression for intensity of sound with the wavelength $\lambda \gg r_0$:

$$I = (\pi/8)\, \rho_0\, (\rho/\rho_0)^2\, r_0\, v_0^2\, \omega\, I_1^2(k\, r_0). \tag{15}$$

It should be noted, taking into account Eq. (14) and $\omega = kv_0$, that $I \sim (R - 1)^{1.5}$. Let us estimate quantitatively the results obtained. As the initial data we take the following typical values of the parameters: the discharge radius $r_0 = 0.15$ cm, the absorption coefficient $\mu \sim 10^{-2}$ cm^{-1}, the temperature $T_0 \sim 16000$ K. With these parameters at hand, the sound velocity in plasma is $c_s = 2{\cdot}10^5$ cm/s, the thermal conductivity $\varkappa \simeq 2.5{\cdot}10^{-2}$ W/cm·K, the coefficient $B \simeq 0.1$, the ratio $\rho/\rho_0 \simeq 2{\cdot}10^{-2}$ (Refs. 1 and 2). Assuming $kr_0 \sim 0.7$ (corresponds to the maximum of function (5)), we calculate the wave number of a wave in plasma $k \sim 5$ cm^{-1}. Substituting these data in Eqs. (11) and (14) and using known representations of Bessel functions,[8] we calculate the threshold power $P_c = \pi r_0^2\, \omega_c \simeq 1$ MW and the front velocity $v_0 = 25$ m/s (at $R = 1.1$). The coefficient of wave convective amplification in plasma can be determined using formula (5). Taking into account the fact that $\alpha \simeq 10^{-9}$ J/cm^2, (Ref. 9) we have $g \simeq 0.1$ cm^{-1}. Before proceeding to the calculation of the level of sound intensity, we have to assess applicability of linear description to acoustic waves. Note that the wave amplitude $a \sim v_0/\omega$, the length of acoustic wave $\lambda = 2\pi c_0/\omega$, the frequency $\omega = kv_0(\sim 10^4\ \mathrm{s}^{-1})$, note also that the condition of applicability of Eq. (15) is fulfilled since $a \ll \lambda$ and $\lambda \gg r_0$. At typical values of the discharge parameters from Eq. (15) the acoustic wave intensity can be calculated as $I \sim 10^{-2}$ W/m^2.

5. Within the limits of hydrodynamic equations in the Boussinesq approximation (variation of entropy of the next order of smallness by the parameter v/c_s) we have studied the process of the light-induced discharge formation. It is shown that the system has the bifurcation point μw_c, which reflects the qualitative changes in the system state and determines the threshold character of the discharge development. When exceeding the energy input threshold, the irreversible processes in the discharge nonequilibrium plasma initiate the discharge self-organization and formation of coherent structures in the form of the entropy-vortex wave. Demonstration of these processes is the macroscopic effects like the ordered motion of the discharge and induced sound. It should be noted that we deal with the cyclic causality: the order parameter (the discharge rate v_0), on the one hand, governs the oscillations (the frequency $\omega = kv_0$), and, on the other hand, the above parameter turns out to result from the inner cooperative space–time structure. This conclusions agrees with the general synergetic concepts,[10] and the situation shows the so-called holistic character of the discharge evolution. The results obtained within the framework of this model both quantitatively and qualitatively agree with the known data. In particular, the investigation into energetics of acoustic emission of the discharge extends the capabilities of remote diagnostics of the integral characteristics of laser beams propagating in the atmosphere.

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