DYNAMIC COMPENSATION FOR LIGHT BEAM THERMAL BLOOMING IN THE TURBULENT ATMOSPHERE BY SIMPLEX METHOD

I.V. Davletshina and S.S. Chesnokov

M.V. Lomonosov State University, Moscow

Received March 19, 1996

This paper is devoted to numerical analysis of the stochastic problems of atmospheric optics. The effect of wind velocity pulsations and large-scale refractive index fluctuations on the energy characteristics of light beams is studied based on simple models of atmospheric turbulence. The feasibility of dynamic compensation for light beam nonlinear and turbulent distortions using the simplex method are presented.

The problem on improvement of the characteristics of laser beams propagated through the atmosphere is of special interest for modern optics and its applications since high-efficiency potentialities of information, technological and measuring systems are essentially limited by the conditions of light wave propagation in real media. The influence of the atmosphere strongly manifests itself in the extended optical channels, for example, along the ground horizontal and slant paths. In this case the propagation of high-power radiation is accompanied not only by nonstationary thermal blooming, under conditions of wind velocity pulsations, but also by the refraction at large scale refractive index fluctuations.

The technique of numerical simulation of nonstationary wind refraction is currently well developed. In Refs. 1–3 a model of wind velocity pulsations is proposed, which makes it possible to describe, in the simplest way, the influence of large-scale turbulent vortices on the beam propagation along near ground paths. In Refs. 4–5 turbulent broadening of a beam and its thermal distortions are considered as additive effects. It is clear that the wind velocity pulsations and refractive index fluctuations jointly affect the structure of thermal channel induced by a beam, therefore for reliable prediction of the beam distortions these effects should be taken into account simultaneously. It should be noted that up to now no detailed studies have been published where such an account is successful.

It happened so that for many years the problem on the development of algorithms of adaptive control of light beam has not been studied theoretically. The gradient method, being originally a basis for the aperture sounding systems, is still the only algorithm used in practice. However, in real systems for atmospheric optics, the fluctuations of radiation and medium parameters, restrict the applicability of the gradient method.[6] This is mainly connected with the large errors when measuring, in real time, the gradients of an optimized functional. For this reason it is interesting to consider direct methods of nonlinear optimization. One of those is the simplex method whose applicability to the problems of laser radiation adaptive focusing in a nonlinear media is analyzed in our earlier papers.[7–12]

This paper is devoted to further development of the simplex method and to analysis of its efficiency in the stochastic problems of atmospheric optics taking into account transient processes in the “beam–medium” system occurring both at variations of a controlled wave front and fluctuating wind velocity and large scale refractive index fluctuations.

1. MATHEMATICAL MODEL OF LIGHT BEAM PROPAGATION IN NONLINEAR TURBULENT MEDIUM

The description of light beam propagation in the atmosphere is conventionally based on the quasi-optical approximation of the diffraction theory, which is considered together with the material equation for perturbations of the medium temperature. The corresponding set of equations in the dimensionless form is written as

$$2\, i \frac{\partial E}{\partial z} = \Delta_{\perp} E + \tilde{n}\, E + R_0\, TE\ , \tag{1}$$

$$\frac{\partial T}{\partial t} + V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} = |E|^2\ , \tag{2}$$

where E is the complex amplitude of the electric field of a light wave; T is the temperature perturbation induced by a beam; $\tilde{n}$ is the random field of the atmospheric refractive index; V_x and V_y are the projections of wind velocity on the coordinate axes $0X$, $0Y$. In Eqs. (1) and (2) standard normalization of variables is used,[13] the nonlinearity parameter

$$R_0 = \frac{2k^2\, a_0^3\, \alpha\, I_0}{n_0\, \rho\, C_p\, V_0} \frac{\partial n}{\partial T} \tag{3}$$

is determined from the average wind velocity V_0.

0235-6880/96/11 936-07 $02.00

We consider the laser beam propagation along the horizontal path above a uniform underlying surface. In this case the random wind velocity field $\mathbf{V}(x, y, z, t)$ is statistically stationary. Since the vertical component of wind velocity V_y in the atmospheric boundary layer is small as compared to the horizontal V_x (Ref. 14), it is assumed that the average velocity is parallel to the surface. It has been known that the outer scale of turbulence L_0 is comparable by the order of magnitude with the height h_0 above the underlying surface, and the spectral maximum of fluctuations of the velocity vertical component is in the frequency range from the interval $0.1V_0/h_0 \le \nu \le V_0/h_0$. Thus, the characteristic period of the velocity pulsations $T_V = 1/\nu$ can be determined as $T_V \approx h\tau_V/a_0$, where $\tau_V = a_0/V_0$ is the convective time.

When using the proposed model on a computer, the velocities $\mathbf{V}$ on the phase screens are represented as a vector sum of the regular component $\mathbf{V}_0$, being parallel to the axis $0X$, and the fluctuation components δV_x, δV_y, following the normal centered distribution law with the variance σ_V^2. The wind velocity pulsations are imitated with a stepwise change of situations in a typical time T_V.

The refractive index fluctuations $\tilde{n}$ are simulated on the basis of a modal approach,[15–19] whereby the instantaneous random phase field $\varphi(\rho, t)$ on the screen are represented as superposition of Zernike orthogonal polynomials $Z_j(\rho/R, \vartheta)$ with the random coefficients $\alpha_j(t)$:

$$\varphi(\rho, \vartheta, t) = \sum_{j=1}^{J} \alpha_j(t)\, Z_j(\rho/R, \vartheta), \qquad (4)$$

where $\boldsymbol{\rho} = \{x, y\} = \{\rho \cos\vartheta, \rho \sin\vartheta\}$; R is the radius of the phase screen.

”ased on the estimates[15] it is clear that for the reliable simulation of random walks and the beam turbulent broadening we need only five Zernike polynomials (subtracting the piston mode), i.e., tilts, defocusing and astigmatism of the wave front. The technique developed in Refs. 17 and 19 enables us to obtain time spectra and autocorrelation functions of random weighting factors $\alpha_j(t)$ and to construct a dynamic model of large scale phase distortions for an arbitrary spectrum of atmospheric turbulence. In this paper we use Karman model of turbulence with the spatial spectrum of the form:

$$\Phi_n(\kappa) = 0.033\, C_n^2\, (\kappa^2 + \kappa_0^2)^{-11/6}, \qquad (5)$$

where C_n^2 is the structure constant of the refractive index fluctuations; κ is the spatial frequency; $\kappa_0 = 2\pi/L_0$.

In accordance with the principle of modal control, the wave front $U(x, y, t)$ of a beam at the transmitting aperture (in the plane $z = 0$) is given as a superposition of the preset optical modes. Starting from the structure of the beam phase distortions in the wind refraction mode, $U(x, y, t)$ is selected in the form

$$U(x, y, t) = \theta_x(t)\, x + \theta_y(t)\, y + S_x(t)\, \frac{x^2}{2} + S_y(t)\, \frac{y^2}{2} + S_{xy}(t)\, 2xy\,. \qquad (6)$$

The control quality is estimated on the basis of the focusing test

$$J_f(t) = \frac{1}{P_0} \iint \sigma(x, y) |E(x, y, z_0)|^2\, \mathrm{d}x\, \mathrm{d}y\,, \qquad (7)$$

where P_0 is the total power of the beam; σ is the aperture function describing the region of the light field localization on the target; z_0 is the path length. In the nonstationary problems, we can use the criterion of the relative control efficiency $\eta(T) = W(T)/W_0(T)$, where

$$W(T) = \int_0^T J_f(t)\, \mathrm{d}t \qquad (8)$$

is the energy of a beam under control, coming into the aperture σ during the time T; W_0 is this same value in the absence of control.

In this paper we describe the study of propagation of the Gaussian beam of the initial radius a_0 =10 cm at the height h_0 = 1 m above the underlying surface. The other parameters of the numerical experiment are the following: the infrared radiation wavelength λ = 10. 6 μm, the mean wind velocity V_0 = 1 m/s, the path length z_0 = 3000 m that makes $0.5\, ka_0^2$, the receiving aperture radius $\sigma(x, y)$ in the observation plane is $r_t = 2a_\mathrm{d}$, where a_d is the radius of diffraction-limited focal spot in vacuum.

2. NONLINEAR DISTORTIONS OF LIGHT BEAMS IN THE PRESENCE OF WIND VELOCITY PULSATIONS AND REFRACTIVE INDEX FLUCTUATIONS

First we consider the simplest turbulent model for analyzing the influence of atmospheric inhomogeneities on the light beam propagation under conditions of wind refraction. It is assumed that the main contribution from large scale atmospheric vortices manifests itself in the wind velocity pulsations. Therefore we start considering the beam propagation in a medium with the wind velocity pulsations along the path, described by Eqs. (1) and (2) ignoring natural fluctuations of the refractive index ($\tilde{n} \approx 0$). It is assumed that the mean time of the pulsations freezing equals $T = 5\tau_V$ what makes, for the selected parameters of the numerical experiment, 0.5 s. Such a frequent change of the medium state enables one to determine the main regularities of the beam propagation and its wave front control at comparatively small computer costs.

In the wind velocity pulsation regime, the transient processes in "the beam–medium" system occur not only at the initial moment of the laser emission turn on, but also when changing the state of the medium. This is caused by the fact that the variation of wind velocity results in considerable variations of the medium effective nonlinearity along the propagation path and, as a consequence, in fluctuations of field parameters at an object it is focused on.

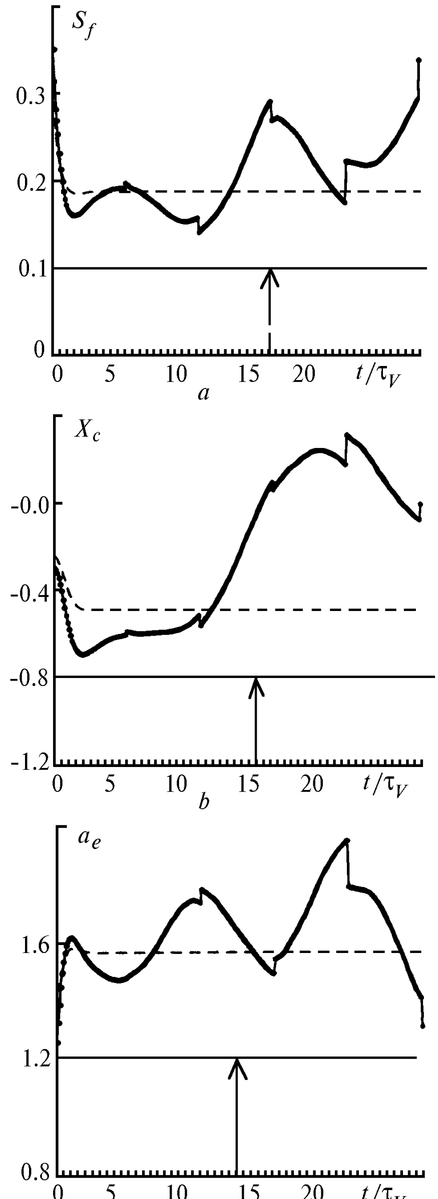

FIG. 1. Typical dependences of the focusing criterion (a), shifts of the beam energy center (b), energy radius (c) on time. Solid lines are for the medium with wind velocity pulsations, dashed lines denote the regular medium. The propagation conditions are characterized by the following parameters: $R_0 = -20$, $\sigma_V = -0.3V_0$, $T_V = 5\tau_V$.

Figure 1*a* presents typical dependences of the focusing criterion on time. The corresponding dependences of the efficient beam width and the shift of its center of gravity on time are presented in Figs. 1*b* and 1*c*.

The presence of wind velocity pulsations along the propagation path results in a larger beam broadening and its random walks in the observation plane (for a comparison, Fig. 1 gives the variation of the beam parameters in a regular medium denoted by dashed lines). The beam behavior of this kind is a result of nonuniform heating of the medium. Figure 2*a* presents an example of the temperature field distribution along the propagation path. For a comparison Fig. 2*b* gives the temperature field induced by a beam in a stationary medium.

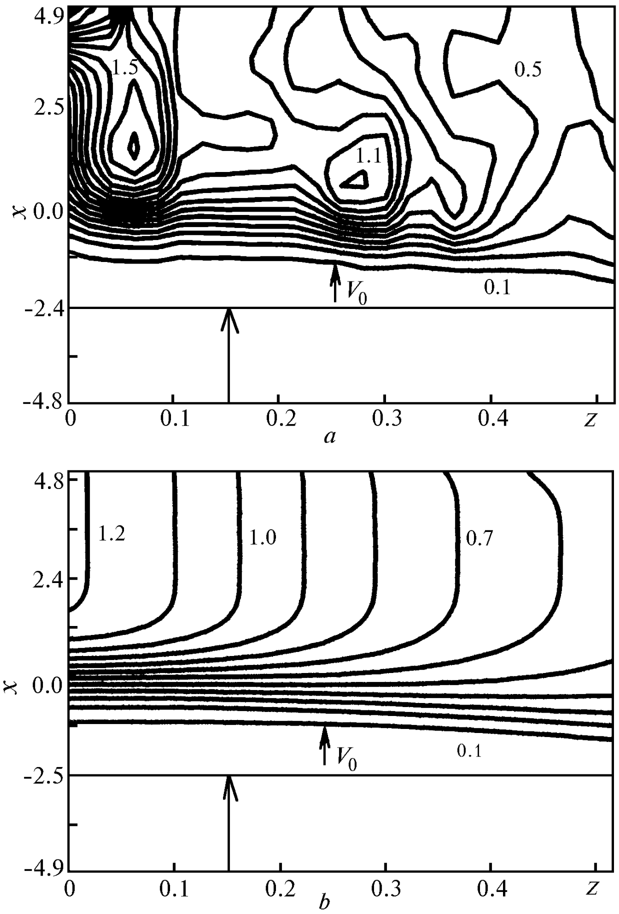

FIG. 2. Lines of equal temperature on the plane XZ at time $t = 7\tau_V$: *a) in the presence of wind velocity pulsations* ($R_0 = -20$, $\sigma_V = -0.3V_0$, $T_V = 5\tau_V$); *b) in a stationary medium. Cross sections of the temperature field are given with the step of* 0.1.

Figure 3 shows the dependences of mean values and standard deviations of the focusing criterion, shift of the beam energy center and its energy radius on the variance of wind velocity fluctuations. These data are obtained using numerical simulation of the propagation of a collimated beam during $T = 25\tau_V$ and averaging over 100 situations. It is clear that with the increase of σ_V the mean values of the focusing criterion and the beam energy radius continue to grow.

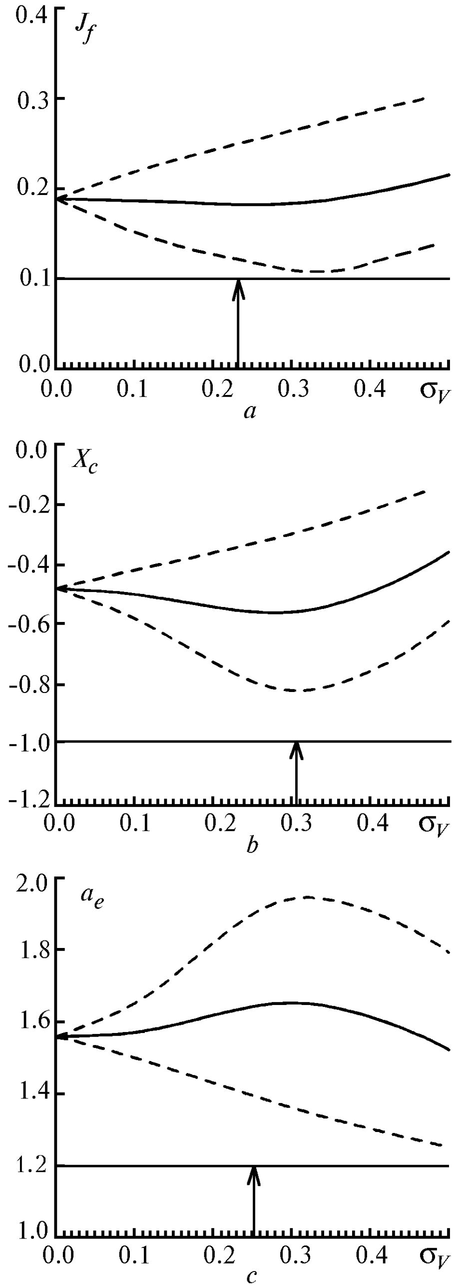

FIG. 3. The influence of the variance of wind velocity fluctuations on the mean values of the focusing criterion (a), shifts of the beam energy center (b), and energy radius (c). Standard deviations of given characteristics are denoted by dashed lines. The propagation conditions are characterized by the following parameters: $R_0 = -20$, $T = 25\tau_V$, $T_V = 5\tau_V$.

In the real atmosphere, large-scale vortices result not only in the wind velocity pulsations but in the refractive index fluctuations as well. Therefore, of interest is the investigation of joint influence of these factors on the beam statistical characteristics and focusing criterion. This problem is described by the set of equations (1) and (2) with the account for all terms.

Table I presents the mean values of the focusing criterion, the beam energy center shift and its energy radius depending on C_n^2 and the variance of wind velocity fluctuations at $R_0 = -20$. The data have been obtained by means of numerical simulation of the collimated beam propagation during $T = 25\tau_V$ and averaged over 100 situations.

TABLE I. Average values of the focusing criterion, shift of the beam energy center, and energy radius. The conditions of propagation: $R_0 = -20$, $T = 25\tau_V$, $T_V = 5\tau_V$.

σ_V/V_0	C_n^2, cm$^{-2/3}$	$\langle J_f\rangle$	$\langle x_c\rangle/a_0$	$\langle a_e\rangle/a_0$
0	0	0.188	−0.48	1.56
0.1	0	0.186±0.032	−0.50±0.08	1.57±0.08
0.1	$6.4\cdot10^{-15}$	0.188±0.033	−0.48±0.08	1.57±0.09
0.1	$6.4\cdot10^{-14}$	0.193±0.034	−0.43±0.10	1.58±0.09
0.1	$6.4\cdot10^{-13}$	0.209±0.038	−0.27±0.16	1.62±0.11
0.3	0	0.183±0.081	−0.56±0.26	1.65±0.29
0.3	$6.4\cdot10^{-15}$	0.184±0.075	−0.52±0.25	1.65±0.28
0.3	$6.4\cdot10^{-14}$	0.185±0.079	−0.49±0.25	1.68±0.29
0.3	$6.4\cdot10^{-13}$	0.195±0.077	−0.34±0.28	1.70±0.28
0.5	0	0.215±0.091	−0.36±0.23	1.52±0.27
0.5	$6.4\cdot10^{-15}$	0.210±0.093	−0.36±0.26	1.73±0.43
0.5	$6.4\cdot10^{-14}$	0.210±0.093	−0.33±0.26	1.74±0.42
0.5	$6.4\cdot10^{-13}$	0.210±0.092	−0.21±0.31	1.75±0.41

Analysis has shown that the combined effect of the refractive index fluctuations and wind velocity pulsations results, on the average, in a more uniform heating of the medium, and consequently, in the decrease of the beam random walks. In this case the main contribution to the beam broadening comes from wind velocity pulsations. Nevertheless, in specific cases, the large-scale refractive index fluctuations can enhance the transient processes in changing the medium states resulting in a sharp decrease of the focusing criterion. To prevent the loss of the control stability under these conditions, we need to conduct a more comprehensive analysis of the adaptive correction characteristics and to develop an algorithm insensitive to sharp changes of the medium states.

3. COMPENSATION FOR TURBULENT DISTORTIONS OF HIGH-POWER LIGHT BEAMS BASED ON THE SIMPLEX METHOD

We start the analysis of the efficiency of dynamic compensation for high-power light beam distortions in the atmosphere with the simplest model, allowing for the wind velocity pulsations. Ignoring the refractive index fluctuations, on the basis of the character of nonlinear beam distortions in a medium with wind velocity pulsations, the controlled wave front could be presented by Eq. (6). However, with a considerable body of coordinates to be controlled it is difficult to perform *a priori* analysis of a trajectory of searching for optimum, being very useful, in particular, for determining the initial simplex configuration. This is especially important in the presence of transient processes since the first steps of the search must be in the correct direction (for example, a beam should start its focusing, and not the reverse). It is difficult to determine this direction in the five-dimensional space. Therefore it is important to try to decrease the number of coordinates to be controlled that simultaneously enables one to increase the response speed of an adaptive system.

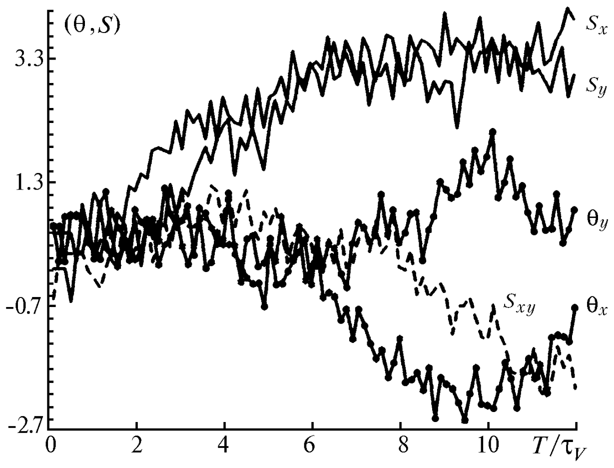

FIG. 4. Dynamics of the coordinates controlled when compensating for the wind refraction using the simplex method. The propagation conditions are characterized by $R_0 = -20$, $T = 12\tau_V$, $T_V = 2\tau_V$, $\sigma_V = 0.3\ V_0$.

Figure 4 presents as an example the dynamics of the coordinates controlled in the basis (6) in the course of dynamic correction for wind refraction in the turbulent atmosphere. It is evident that adaptive correction by the simplex method gives rise to a compulsory beam scanning due to the stepwise variation of the coordinates controlled, especially tilts. Taking into account the fact that in the presence of random wind velocity pulsations beam defocusing is axially symmetric, the number of variables controlled should be decreased by using the combined mode $(x^2/2 + y^2/2)$, i.e., assuming that

$$U(x, y, t) = \theta_x(t)\,x + \theta_y(t)\,y + S(t)\left(\frac{x^2}{2} + \frac{y^2}{2}\right) + S_{xy}(t)\,2xy\,. \qquad (9)$$

Considering that the variable S_{xy} rapidly decreases with time, it may be excluded in some problems. In this case the wave front is of the form:

$$U(x, y, t) = \theta_x(t)\,x + \theta_y(t)\,y + S(t)\left(\frac{x^2}{2} + \frac{y^2}{2}\right). \qquad (10)$$

In the subsequent numerical experiments, the beam control was considered in a time $T = 12\tau_V$ since the turning on the laser source. Efficiency of the search was estimated by the parameter $\eta(T)$. The results of numerical simulation have shown that the optimal size, L, of the simplex can be estimated based on the recommendations given in Refs. 9 and 12.

It should be noted that the modification of the algorithm of simplex search[9] normally used cannot provide stability of the control under pulsation conditions because here the approach of simplex to "drifting" objective is provided. Therefore it occurred so that the search should be used with variable strategy,[11] which can be subdivided into two stages. The first stage is the control at the initial phase of medium heating (during the time $2\tau_V$) with the use of an algorithm with a forced vertex reflection[20] that makes it possible to avoid "recycling" of the simplex. Then, at the second stage, when of fundamental importance are the random beam walks and transient processes, appearing when changing the medium states, an algorithm with free vertex reflection should be used. Its main rule is in the reflection of the worst simplex vertex without any supplementary conditions. As is shown below, such a control organization enables one to compensate for the random mean walks and to avoid the unstable regimes of search.

In the problem considered we assume that the time of freezing of wind velocity pulsations is $T_V = 2\tau_V$. Although this regime is unrealistic, it enables us to study in detail the simplex method algorithm stability in stochastic problems at acceptable calculation costs. Table II shows the control efficiency based on the use of the simplex method in the bases (6), (9), (10). The values of $\eta(T)$ presented have been obtained in one and the same situations of the wind velocity pulsation distribution along the path and averaged over 20 events. From the table values we see that the control efficiency in different bases is determined by such factors as the nonlinearity parameter and the wind velocity variance. For example, in the case of weak nonlinearity $\langle |R| \rangle \le 10$ the three-dimensional basis (10) reveals considerable advantages at $0.1 \le \sigma_V/V_0 \le 0.5$. At a moderate nonlinearity $(20 \le \langle |R| \rangle \le 30)$ the control efficiency in the bases (6) and (9) is the same. Thus, under the considered conditions the four-dimensional basis (9) is sufficient and there is no need to use a five-dimensional one (6).

It should be noted that the mean values of the relative control efficiency parameter $\eta(T)$ increase with the growth of both the medium nonlinearity and the variance of wind velocity fluctuations. It is evident, as in the case of stationary medium, this occurs because of a more uniform heating of the propagation channel due to temperature mixing achieved by scanning with a controlled beam. This effect also influences the statistical characteristics of the beam and the goal function being optimized. Under conditions of wind velocity pulsations in the range $0.1 \le \sigma_V/V_0 \le 0.5$, the simplex search algorithm is stable, and with the increase of σ_V/V_0 the standard deviation of the focusing criterion does not grow. Evidently, this is explained by the fact that the algorithm used provides steady beam scanning in the mutually perpendicular planes. As a result, the mean shift of the beam center of gravity does not exceed $a_0/2$ within a wide range of variation of the nonlinearity parameter $20 \le \langle|R|\rangle \le 30$; in this case, for the values $20 \le \langle|R|\rangle \le 30$, the control based on the simplex method enables one to increase the energy $W_0(T)$ over the control period by 1.5 times, on the average, as compared with the cases of propagation of a collimated or focused beams.

TABLE II. Relative control efficiency $\eta(12\tau_V)$ depending on the nonlinearity parameter and the variance of wind velocity pulsations.

Control basis and the number of variables	$\|R_0\|$								
	10			20			30		
	σ_V/V_0								
	0.1	0.3	0.5	0.1	0.3	0.5	0.1	0.3	0.5
(6) $N = 5$	1.34	1.37	1.33	1.49	1.45	1.38	1.55	1.56	1.61
(9) $N = 4$	1.38	1.51	1.46	1.50	1.52	1.47	1.46	1.58	1.68
(10) $N = 3$	1.54	1.58	1.56	1.49	1.49	1.50	1.51	1.61	1.65

Let us now consider the problem on control of a beam propagating in a randomly inhomogeneous nonlinear medium, described by Eqs. (1) and (2) with regard for all terms. The correction efficiency is estimated, as previously, based on the normalized total light energy coming to a receiving aperture during the control time T. First we need to determine the manner in which the control basis dimension influences its efficiency. The results of numerical simulation of the light beam phase control in this problem are presented in Table III.

TABLE III. Average values and standard deviations of the control efficiency $\eta(12\tau_V)$. The conditions of propagation are characterized by $R_0 = -20$, $\sigma_V/V_0 = 0.3$, $T_V = 2\tau_V$.

Parameter	Control basis			
	(6) $N = 5$	(9) $N = 4$	(10) $N = 3$	$U = 0$, $N = 0$
σ_η	0.34	0.39	0.46	0.46
$\langle\eta\rangle$	1.60	1.57	1.68	1.00

The values of the relative control efficiency, calculated in the interval $T = 12\tau_V$, are averaged over 10 realizations. It is clear that the most effective control is in the three-dimensional basis although five-dimensional basis enables one to achieve less spread of energy coming to the receiving aperture. As a whole, one can state that the phase correction based on the simplex method is efficient in a wide range of the parameter C_n^2. This is confirmed by the results of numerical simulations given in Table IV.

TABLE IV. Average values of the focusing criterion, the shift of the beam energy center, and energy radius when controlling based on the simplex search algorithm. The conditions of propagation are characterized by the parameters $R_0 = -20$, $T = 25\tau_V$, $T_V = 5\tau_V$.

σ_V/V_0	C_n^2, $cm^{-2/3}$	$\langle J_f\rangle$	$\langle x_c\rangle/a_0$	$\langle a_e\rangle/a_0$
0.1	$6.4\cdot10^{-14}$	0.33±0.06	−0.30±0.15	1.36±0.05
0.1	$6.4\cdot10^{-13}$	0.32±0.02	−0.27±0.12	1.37±0.05
0.1	$6.4\cdot10^{-12}$	0.31±0.02	−0.27±0.16	1.36±0.05
0.3	$6.4\cdot10^{-14}$	0.330±0.02	−0.27±0.15	1.36±0.05
0.3	$6.4\cdot10^{-13}$	0.32±0.03	−0.25±0.16	1.35±0.04
0.3	$6.4\cdot10^{-12}$	0.33±0.03	−0.21±0.12	1.32±0.04
0.5	$6.4\cdot10^{-14}$	0.32±0.02	−0.33±0.12	1.50±0.03
0.5	$6.4\cdot10^{-13}$	0.31±0.02	−0.33±0.11	1.51±0.04
0.5	$6.4\cdot10^{-12}$	0.32±0.03	−0.30±0.10	1.53±0.03

CONCLUSION

This paper describes the analysis of propagation of high-power light beams in the turbulent atmosphere, which makes it possible to conclude that:

1. Wind velocity pulsations along the path, caused by the large scale atmospheric vortices, result in the improvement of propagation conditions as compared with the case of stationary medium, despite of the increase of the root-mean-square beam radius with the growth of the wind velocity fluctuation intensity. The observed decrease of the mean shift of its center of gravity results, on the average, in the increase of the light field concentration on the target. Thus, with the increase of standard deviation of wind velocity from zero up to the half of mean value of wind velocity, the focusing criterion, averaged over the time and realizations, increased by 10 or 15%. Simultaneously with this the standard deviations of the focusing criterion, the shifts of the beam center of gravity and its radius increased by a factor of two or three.

2. Within a wide range of values of the structural constant C_n^2, the large scale refractive index fluctuations, occurring simultaneously with the wind velocity pulsations, do not practically result in the variation of the light field mean concentration on the target causing a slight increase (up to 10%) of statistical variance of the focusing criterion and a

marked increase (by 10–60%) of the variance of the shift of the beam center of gravity and its rms radius.

3. Adaptive compensation for high-power light beam distortions in the turbulent atmosphere based on the simplex method with optimized parameters makes it possible to increase the mean light energy coming to the target, as compared with the case of uncontrolled beam (collimated or focused), on the average, by 70 or 80%. This effect is mainly due to a decrease in the controlled mean beam shift (by 20–60%) and its energy radius (by 15–30%). Simultaneously in the control we observe a considerable decrease of variances of the beam energy parameter fluctuations.

ACKNOWLEDGMENT

The present work has been performed under the support from the International Science Foundation and Russian Foundation for Fundamental Research (Grant NCR300).

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