PHASE CONJUGATION AND INSTABILITY IN AN INTERACTION OF GAUSSIAN DIFFRACTING LIGHT BEAMS PROPAGATING IN OPPOSITE DIRECTIONS THROUGH MEDIA WITH THE KERR NONLINEARITY. II. NUMERICAL EXPERIMENT

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Efficiency and quality of phase conjugation in a four-wave interaction of noncollinear diffracting light beams propagating in opposite directions are analyzed. Development of non-stationary (periodic) variations of the interaction characteristics is established. A nature of their occurrence is discussed. It is shown that the basic role in their development is played by self-action of light beams. The correctness of the obtained results is estimated with the help of spectral analysis of the characteristics of interacting waves.

1. INTRODUCTION

In Part I of the present paper development of convective instability in a system of counterpropagating noncollinear beams has been analyzed. In linear approximation conditions have been specified, at which such instability takes place. At the same time it is well known that in a four-wave interaction (FWI) under conditions of strong nonlinear interaction of light beams oscillating modes may appear, caused by other physical factors, for example, by mutual transfer of light energy.

In Part II of the paper the method of numerical modeling analyzes the influence of physical factors (self-action, light energy transfer) and noncollinearity of beams on occurrence of absolute instability. Its mechanism is also studied.

2. RESULTS OF NUMERICAL EXPERIMENTS

In the course of numerical experiments we are interested in evolution of the the beam center position

$$X_C(z, t) = \int_0^{L_X} (x - X_C(0, t)) \times$$

$$\times |A(z, x, t)|^2 \, dx / P(z, t);$$

$$P(z, t) = \int_0^{L_X} |A(z, x, t)|^2 \, dx, \quad (1)$$

quality of phase conjugation (PC)

$$\chi = \sqrt{\int_0^{L_X} |A_3 A_4|^2 \, dx / P_3 P_4 \Big|_{Z=0}}\,, \quad (2)$$

the maximum intensity in the beam cross section L_Z

$$I_m(z, t) = \max_X |A_m(z, x, t)|^2, \quad (3)$$

and its transverse coordinate $X_m(z, t)$, the power reflectance

$$R_P = P_4(0, t)/[P_3(0, t)], \quad (4)$$

and the maximum intensity reflectance

$$R_{I_{max}} = I_{m,4}(0, t)/[I_{m,3}(0, t)]. \quad (5)$$

Because in the course of numerical experiments it was established that the interaction has essentially non-stationary character and both complex amplitudes of interacting waves and characteristics of interaction introduced above oscillated with various spatiotemporal scales, much attention was given to their spectral analysis to check the accuracy of the obtained results. The necessity of such checking follows from the obvious fact that the solution obtained on spatiotemporal grids with the characteristic scales τ and h adequately describes the solution of initial differential problem, in particular, under condition that in the course of calculations the amplitudes of spatiotemporal harmonics with numbers exceeding a given value N_0 are all equal to zero. In other words, the filtering action of a difference grid should not distort a spectrum of the solution to a differential equation. With this purpose the numerical solution and the characteristics of interaction listed above were represented as spatiotemporal Fourier series of the form

$$f(m) = \sum_{n=0}^{N} a_n \cos \frac{\pi\, nhm}{L} + b_n \sin \frac{\pi\, nhm}{L}\,,$$

$$0 \le m \le M, \quad (6)$$

where a_n, b_n are the expansion coefficients, M is the maximum number of harmonics, $2 * L$ is the dimension of the appropriate area along the spatiotemporal expansion coordinate, and h is the step of the grid on it.

Criterion of the adequate description of occurring processes is approaching to zero of the expansion coefficients $a_n \to 0$ $(b_n \to 0)$, starting from a fixed value $n = N_0$. In this case, these coefficients are calculated from the formulas

$$a_n(b_n) = \frac{2}{L} \sum_{m=0}^{N} f(m) \cos \frac{\pi\, nhm}{L} \left(\sin \frac{\pi\, nhm}{L} \right). \quad (7)$$

Here, $f(m)$ is the value of the function in point m of the grid.

Our numerical experiments were carried out for the following values of the parameters:

$$\gamma = -12, -17, -20;\ L_X = 14;\ L_Z = 0.25;$$

$$R_0 = 1;\ R_m = 10;\ \beta = 0.05;\ D = 0.1. \quad (8)$$

Taking into account above-stated, we selected the following number of nodal points along the transverse (N_X) and longitudinal (N_Z) coordinates:

$$N_X = 650,\ N_Z = 81. \quad (9)$$

In this case the amplitudes of harmonics with large numbers can be taken zero with high accuracy in the examined range of variations of the nonlinearity factor, which provided the adequate description of evolution of spatiotemporal spectra of interacting waves.

The purpose of our experiments is to estimate contributions of beam noncollinearity, self-action, and energy transfer to development of oscillating modes of variations of the interaction characteristics.

For numerical modeling of Eqs. (1) and (2) in Part I we used nonlinear conservative difference schemes[1,2] of the second order of accuracy on spatiotemporal variables.

We note that, as modeling of interaction of four waves propagating in the same directions has demonstrated, the backward energy transfer (from signal and backward waves to the pump waves) occurs, if the phase run-on exceeds 3–4 units of nonlinear lengths. In the situation examined here, up to $\gamma = -10$ and $\beta = 0$, the phase run-on does not exceed 2.5; therefore, the backward energy transfer affects development of free oscillations for the parameters (8) at $|\gamma| > 16$.

We first analyze temporal evolution of the maximum intensity of the conjugated wave at the exit from the medium $I_m(0, t)$ (Fig. 1).

In Figs. 1*a*, *b*, and *c* dynamics of the maximum intensity variations is shown for three values of the beam power. We note that for the least $|\gamma|$, a stationary value of maximum intensity is established after a transient process. Moreover, in an interaction of noncollinear beams a greater output intensity is reached. As the initial beam power increases up to $|\gamma| = 17$, the oscillations develop. In case of slant beam incidence oscillations develop earlier. As the beam power increases further, the oscillations become complex in character: several periodic processes are present in them. The oscillation frequency increases with time.

To establish the reason of oscillation development, Figs. 1*d* and *e* show dependences of the reflected wave intensity for model of an interaction of light beams without self-action (terms F_{sj} in Eq. (1) of Part I are omitted). Comparison of these figures with Figs. 1*a*, *b*, and *c* shows that the basic mechanism of oscillation occurrence in FWI is energy transfer among the interacting waves. Moreover, it should be noted that noncollinearity of interacting beams results in the decrease of the oscillation amplitude and frequency of oscillations of the intensity maximum. We emphasize that in an interaction of collinear beams for $|\gamma| = 17$ a bistable temporal dependence of the maximum intensity is realized (see Fig. 1*d*).

Conclusions about the most essential factor influencing occurrence of instability are also confirmed by the results of calculations without considering energy transfer among interacting waves (terms F_{cj} in Eq. (1) of Part I are omitted). Figures 1*f* and *g* show a significant decrease in the oscillation frequency of output intensity of the reflected beam. However, noncollinearity of interacting beams in this case results in stronger oscillations in comparison with an interaction of counterpropagating beams.

Analogously behave the other characteristics of interaction – PC quality and power and intensity reflection coefficients. As an example, Fig. 2 shows dynamics of PC quality. As follows from an analysis of the figure, noncollinearity of interacting beams results in the increase of oscillation amplitude and frequency of this characteristic. We note that without energy transfer among the interacting beams (simply counter propagation) their noncollinearity does not influence the quality of phase conjugation. In case of absence of self-action (when the length of interaction is much greater than the nonlinear length) in an interaction of collinear beams the quality oscillations may practically vanish (Fig. 2*c*): their amplitude in a quasi-stationary mode is very small.

At the same time, noncollinearity of interacting beams results on the initial stage in the increase of oscillation amplitude. Then χ undergoes faster PC quality variations and exhibits higher average level of its oscillations.

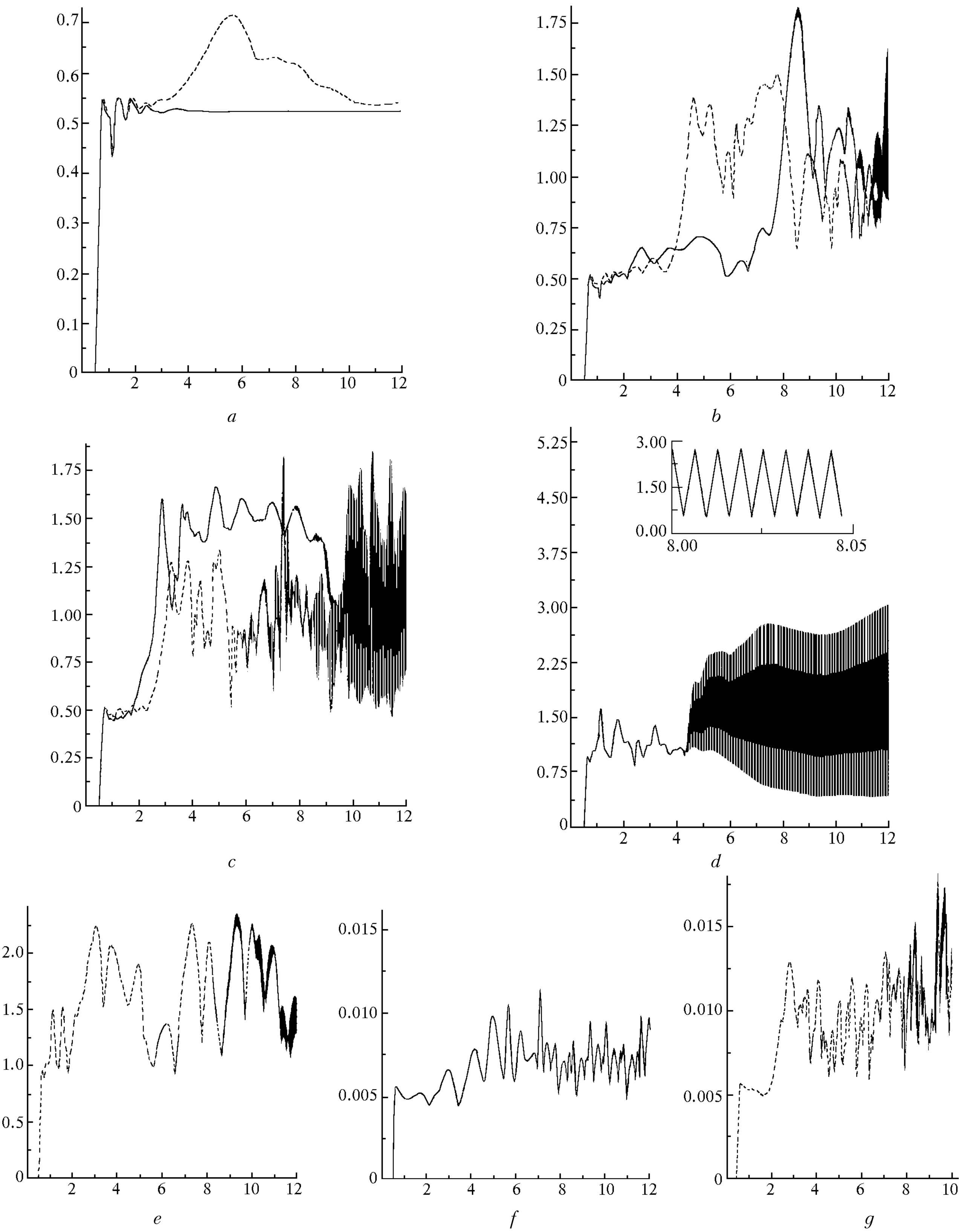

FIG. 1. Dynamics of variations of the reflected wave intensity at the exit from the nonlinear medium in an interaction of collinear (solid curve) and noncollinear ($\beta = 0.5$) (dotted curve) beams at $\gamma = -12$ (a), -17 (b), and -20 (c) considering self-action and energy transfer; curves d and e are for collinear (d) *and noncollinear ($\beta = 0.5$) (e) beams without self-action for $\gamma = -17$; curves f and g are for an interaction of waves without energy transfer for collinear (f) and noncollinear ($\beta = 0.5$) (g) beams at $\gamma = -17$.*

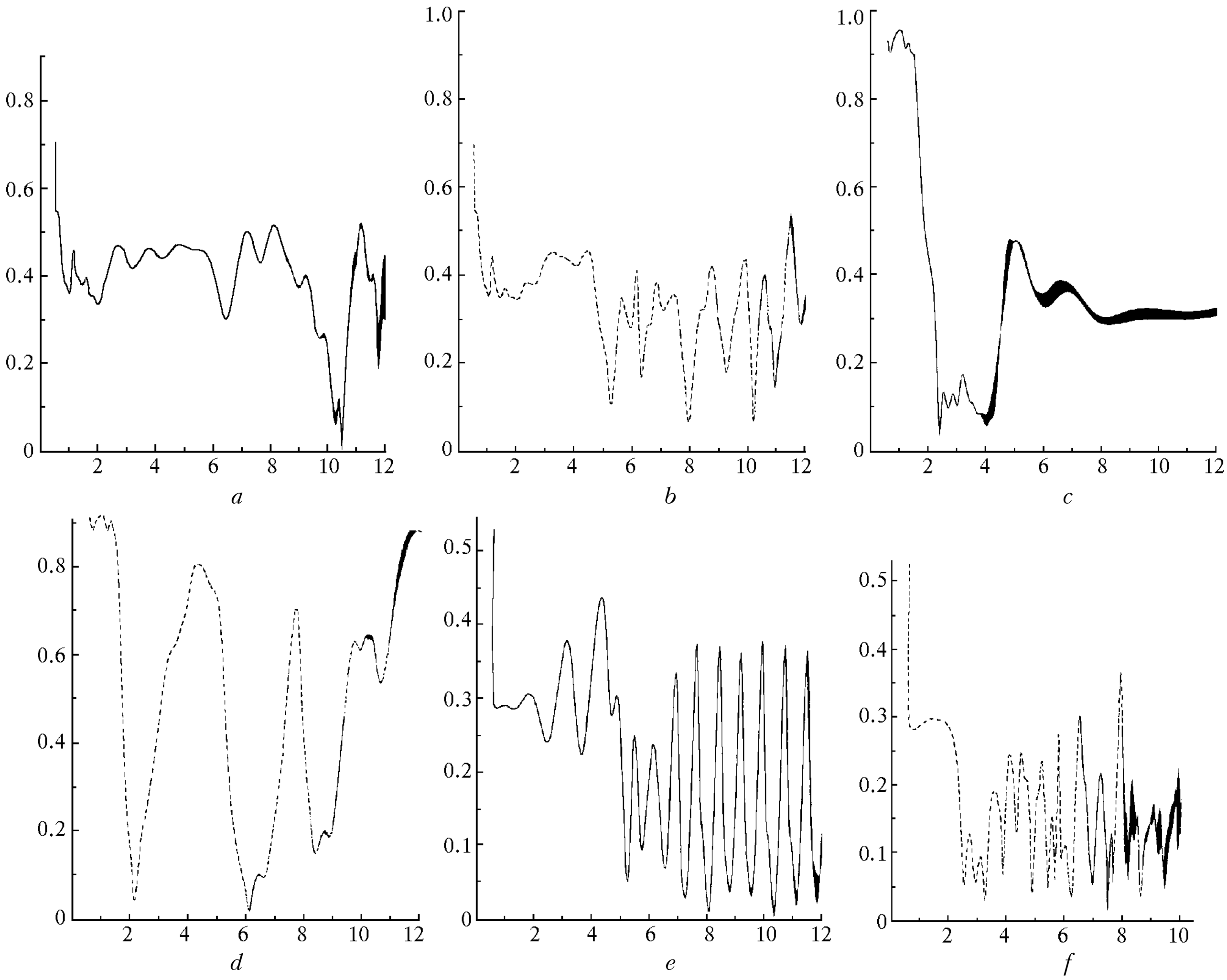

FIG. 2. Dynamics of variations of the PC quality in an interaction of collinear (a) and noncollinear (b) beams at $\gamma = -17$ *considering self-action and energy transfer: cases c and d correspond to an interaction of collinear (c) and noncollinear (*$\beta = 0.5$*) (d) beams without self-action at* $\gamma = -17$*; cases e and f correspond to an interaction of collinear (e) and noncollinear (*$\beta = 0.5$*) (f) beams without energy transfer at* $\gamma = -17$*.*

The variations of power and intensity reflection coefficients also have complex character. The maximum value of Rl reaches 92; its minimum is equal to 70 (for $\beta = 0$) and 75 (for $\beta = 0.5$). The intensity reflectance changes between 50–150 ($\beta = 0$) and 40–185 ($\beta = 0.5$). Hence, the local beam characteristics are more sensitive to geometry of interaction, and noncollinearity of interacting beams may strongly increase them. The integral characteristics are less sensitive. It is important to emphasize that without self-action the intensity reflectance significantly increases in the minimum (up to 100) and the maximum (up to 240) in comparison with phase conjugation with self-action. Moreover, noncollinearity results in amplification of oscillating modes.

In an interaction of noncollinear beams their centers also undergo oscillations (Fig. 3). In this case, when we consider both self-action and energy transfer (Fig. 3*a*), the centers of the third less intense incident beam and of the fourth reflected beam oscillate with smaller frequencies and amplitudes in comparison with cases without self-action (Fig. 3*b*) and without energy transfer (Fig. 3*c*). It should be emphasized that in all three cases the incident beam center may lie above the reflected beam center; moreover, without self-action it lies above the reflected beam center during most of the interaction time. The beam self-action has the greatest influence on oscillations of the beam center of gravity, because in this case the oscillation amplitude and frequency are much higher than when we consider only energy transfer or both factors. In conclusion we note that for the first more intense and the second beams oscillations of their centers of gravity also take place and are even more pronounced; the amplitudes of oscillations of the positions of beam centers may exceed those of less intense beams almost 4 times.

3. CONCLUSIONS

The process of FWI of noncollinear counterpropagating beams has essentially non-stationary character. For its adequate description

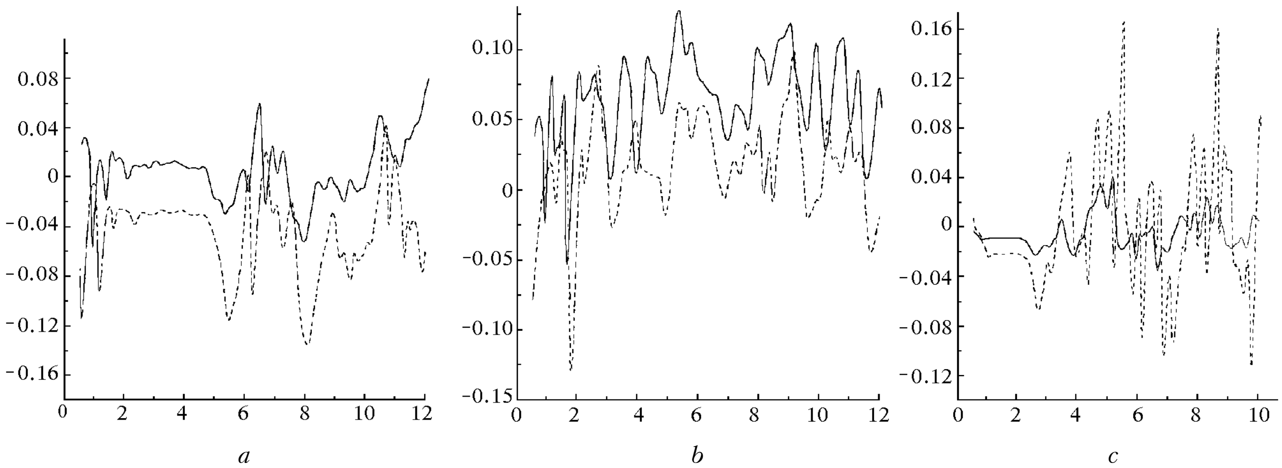

FIG. 3. Dynamics of variations of the position of the incident signal beam center (solid curve) in the cross section $z = L_z$ and of the conjugated beam center (dotted curve) in the cross section $z = 0$ at $\gamma = -17$ and $\beta = 0.5$ for complete variant (a), without beam self-action (b), and without energy transfer (c).

the non-stationary equations should be used. Because in an interaction of counterpropagating beams, as a rule, complex spatiotemporal oscillating processes are developed, for its modeling it is necessary to check the spectral structure of the intensity distribution obtained in numerical experiments.

The basic reason of development of non-stationary processes in FWI is mutual energy transfer among the interacting waves. Beam noncollinearity can not only smooth oscillations because of beam mixing, but also, on the contrary, intensify them.

Development of oscillations of the interaction characteristics and their maximum values are critical to the powers of interacting beams: they occur when the initial pump beam power exceeds a certain characteristic value. In case of weak interaction the energy center of the conjugated beam may lie below the signal beam center.

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