ANALYSIS OF SOME ALGORITHMS FOR MINIMIZING THE ANGULAR DIVERGENCE OF PARTIALLY COHERENT OPTICAL RADIATION

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Received June 26, 1989

The conditions for achieving maximum radiation on target under conditions of "fast" control of the amplitude-phase profile of the beam in the radiation plane are formulated. The numerical schemes for adaptive and program phase control are examined in the aberration-free approximation for a medium with nonlinearity of the wind type. Analysis of the results makes it possible to identify regions where phase correction is very effective and regions where such correction gives only an insignificant gain.

The propagation of high-power laser radiation in the atmosphere is accompanied by different nonlinear effects that distort the spatial and temporal characteristics of the radiation.[1] We shall examine the possibility of compensating the nonlinear distortions by means of phase correction.

Let high-power optical radiation enter a nonlinear medium, and assume that the curvature of the wavefront can be controlled.

The problem is to minimize the angular divergence in the far zone of diffraction after the beam has passed through a thin layer of the nonlinear medium with effective thickness z_s (z is the propagation coordinate). We assume that the angular divergence reaches its minimum value when the intensity at the point of reception is maximum.

We shall study the case of radiation when

$$\tau_{cor} \le \tau_{nl}$$

where τ_{cor} is the correction time of the phase front at the radiation source and τ_{nl} is the nonlinear response time of the medium, whose value was determined in Ref. 2 for a thermal nonlinearity. Satisfaction of this condition means that the phase correction is performed for radiation propagating in a "frozen" refractive medium.

In Ref. 3 it was shown for partially coherent radiation, whose propagation is described by the radiation transfer equation, that if the source is placed at the point of reception, located inside the nonlinear layer, than the wave arriving from it in the initial plane will give the optimal phase front. In other words, we obtain maximum intensity at the point of reception by transmitting a beam that is phase-conjugated with respect to the beam arriving from the point source. For the case when the point of reception is located beyond the nonlinear layer in the far zone of diffraction, the wave emanating from it arrives as a plane wave at the output end of the nonlinear layer. Having passed through the layer the wave arrives in the initial plane with some curvature of the phase front. By constructing the phase-conjugate front for the transmitted beam we obtain maximum intensity at the point of reception. This was proved in Ref. 4 in the aberration-free approximation.

In Refs. 5 and 6 it was proved in the parabolic-equation approximation for the case when the refractive properties of the medium can be described in the phase-screen approximation that maximum intensity at the point of reception is achieved if a front that is the conjugate of the wavefront emanating from the point of reception is formed in the radiation plane.

We shall show that this condition is also satisfied for the case when the phase-screen approximation is not applicable.

We give the field in the radiation plane in the form

$$U(z = 0, \vec{\rho}) = U_0(\vec{\rho}) = A(\rho) \exp(i\vec{\varphi}(\vec{\rho})),$$

where $A(\rho)$ is the fixed amplitude distribution on the field on the radiating aperture S_r and $\varphi(\rho)$ is the controlled phase distribution.

The field at the point of reception can be represented in the form

$$U(z_0, \vec{\rho}_0) = \iint\limits_{S_1} d\vec{\rho}\, U_0(\vec{\rho}) G_+(0, \vec{\rho};\ z_0, \vec{\rho}_0),$$

where G_+ is the Green's function describing the forward propagation of the wave. According to the reciprocity theorem

$$G_+(0, \vec{\rho};\ z_0, \vec{\rho}_0) = G_-(z_0, \vec{\rho}_0;\ 0, \vec{\rho}) = A_-(\vec{\rho}) \exp(i\vec{\Psi}(\vec{\rho})),$$

where G_- is the Green's function describing backward propagation of the wave, and $A_-(\rho)$ and $\Psi(\rho)$ are the distribution of the amplitude and phase of

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the wave emanating from the point (z_0, ρ_0) in the plane $z = 0$.

Then we obtain for the intensity at the point of the reception

$$V(z_0, \vec{\rho}_0) = |U(z_0; \vec{\rho}_0)|^2 =$$

$$= \left| \iint_{S_1} d\vec{\rho}\, A(\vec{\rho}) \exp(i\varphi(\vec{\rho}))\, A_-(\vec{\rho}) \exp(i\psi(\vec{\rho})) \right|^2 . \quad (1)$$

It is obvious that maximum intensity is achieved when the condition

$$\varphi(\vec{\rho}) = - \psi(\vec{\rho}). \quad (2)$$

is satisfied, i.e., when the phase in the radiation plane is the conjugate of the phase of the wave emanating from the point of reception. If the refractive medium has a focusing effect on the radiation, then at the radiating aperture the value of the amplitude A_- of the wave emanating from the point of reception will be greater than the corresponding amplitude of the wave propagating in a uniform medium. Then, as follows from Eq. (1), the intensity at the point of reception will exceed the diffraction limit. Conversely, if the medium has a defocusing effect, then the diffraction limit becomes unattainable for any phase correction.[3, 4]

Next, we shall study the situation when in addition to controlling the phase it is also possible to control the amplitude distribution on the radiating aperture, maintaining the beam power

$$P = \iint_{S_1} d\vec{\rho}\, A^2(\vec{\rho}).$$

constant. The problem is to find an amplitude distribution that maximizes the intensity at the point of reception. As follows from Eq. (1), the intensity reaches its maximum value when, under the condition (2), the integral

$$\iint_{S_1} d\vec{\rho}\, A(\vec{\rho})\, A_-(\vec{\rho})$$

Is maximized or, which is the same thing, the following expression is minimized:

$$\iint_{S_1} d\vec{\rho}\, A^2(\vec{\rho}) + C_1^2 \iint_{S_1} d\vec{\rho}\, A_-^2(\vec{\rho}) - 2C_1 \iint_{S_1} d\vec{\rho}\, A(\vec{\rho})\, A_-(\vec{\rho}). \quad (3)$$

where C_1 is a constant determined from the condition

$$\iint_{S_1} d\vec{\rho}\, A^2(\vec{\rho}) = C_1^2 \iint_{S_1} d\vec{\rho}\, A_-^2(\vec{\rho})$$

Transforming the expression (3) into the form

$$\iint_{S_1} d\vec{\rho} \left[A(\vec{\rho}) - C_1 A_-(\vec{\rho}) \right]^2 ,$$

we find that the intensity at the point of reception is maximized if

$$A(\vec{\rho}) = C_1 A_-(\vec{\rho}).$$

We did not indicate above the equation of which the Green's functions presented are solutions, because we used the most general properties of the Green's functions. This equation can be a parabolic equation as well as the Helmholtz equation.

In a real situation the condition that the medium be ''frozen" cannot always satisfied, so that we shall analyze the possibility of phase correction for a different limiting case, when $\tau_{cor} \gg \tau_{nl}$. We shall perform this analysis in the aberration-free approximation.

The dimensionless beam width $g(z)$ for nonlinearities of any type satisfies in the aberration-free approximation the equation

$$\frac{d^2 g}{dz^2} = f(z) g + \beta g^{-3}. \quad (4)$$

Here and below z is scaled to the refraction length:

$$L_R = \frac{1}{2} |\varepsilon_2(z = 0)|^{-1/2}; \; f(z) = \frac{\varepsilon_2(z)}{|\varepsilon_2(z = 0)|};$$

$$\varepsilon_2(z) = \left. \frac{d^2 \varepsilon(z, \vec{R})}{dR^2} \right|_{R=0},$$

where ε is the perturbation of the dielectric constant, whose functional dependence on the intensity is determined by the type of nonlinearity; $\beta = (L_R/L_D)^2$ where L_D is the refraction length, $L_D = ka_0^2 / \sqrt{1 + (a_0 / a_c)^2}$, a_c is the coherence radius of the radiation, and a_c is the beam radius in the plane $z = 0$.

Let the nonlinearity of the medium be of the wind type. We shall assume that the strength of the nonlinear refraction decreases exponentially as a function of z in the layer. In other words, we shall perform phase correction with

$$f(z) = \exp(-z/z_s) g^{-3}(z), \quad (5)$$

where z_s is the effective thickness of the nonlinear layer.

We shall study the case of adaptive control of the wavefront. At each moment in time a phase front that is conjugated with respect to the wavefront arriving at the point of reception is formed on the radiating aperture. Since $\tau_{cor} \gg \tau_{nl}$ the correction of the phase front lags behind the changes in the distribution of the dielectric constant in the layer of nonlinear medium, and therefore the adaptive-control process is of an iterative character.[7]

We shall model this adaptive control using the following scheme. From the initial plane the collimated beam with width $g = 1$ propagates through the nonlinear medium up to the exit boundary of the medium, and the dielectric-constant distribution created after the passage of the beam is remembered. A plane wave propagating to the initial plane with a fixed distribution of the dielectric constant is given at the exit boundary. This wave arrives, after passing through the nonlinear medium, in the initial plane with some phase front. Here a front that is the phase-conjugate of the arrived front is created. Next, the beam with $g = 1$ and the created phase-conjugated front propagates through the nonlinear medium to the exit boundary of the medium and once again the dielectric-constant distribution created after the passage of the new beam is fixed, and so on.

The angular divergence of the beam in the far zone (in what follows, the angular divergence) is calculated at the exit boundary of the layer based on the obtained beam width and curvature of the phase front of the beam:

$$\gamma = \sqrt{\left[\left.\frac{dg}{dz}\right|_{z=z_s}\right]^2 + \beta g^{-2}(z = z_s)} \ .$$

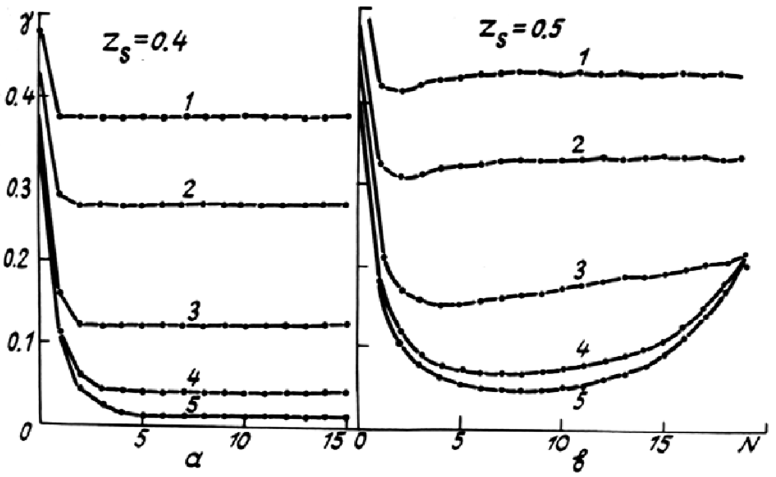

FIG. 1. The angular divergence γ versus the number of iterations N for several values of β (β = $(L_R/L_D)^2$): β = 0.5 (1), 0.05(2), 0.01(3), 0.001 (4), and 0.0001 (5); z_c = 0.4 (a) and 0.5 (b).

Figure 1 shows the dependence of γ on the number of iterations N; this dependence was obtained by numerical modeling of the adaptive control based on the proposed scheme. For $z_s = 0.4$ (Fig. 1a) one can see that as the number of iterations increases γ reaches a stationary level for each value of β presented. Two to five iterations are required to reach this level; in addition, more iterations are requited for lower values of β.

The calculation showed that the given stationary level is close (the difference does not exceed 0.5%) to the minimum angular divergence achieved with optimal focusing of the beam.[4]

We note that for $z_s < 0.4$ the stationary level is already reached after one to three iterations, and once again more iterations are required for smaller values of β. For $z_s = 0.5$ (Fig. 1b) there is no stationary level for the angular divergence, and therefore the iteration scheme of the proposed adaptive control becomes unstable. As the number of iterations increases we obtain a minimum in the dependence of γ on the number of iterations; beyond this minimum γ increases. It should be noted that the obtained minimum gives an angular divergence that is close to minimum, as before. This picture is also observed for $z_s > 0.5$, and in this case the minimum is sharper. Therefore, at some number of iterations, for $z_s \geq 0.5$, this system results in lower efficiency of radiation transfer so that in order for the proposed scheme of adaptive control to work additional monitoring of the achievement of minimum angular divergence or, which is equivalent, achievement of maximum intensity at the point of reception is necessary.

For this reason we shall now consider a different type of phase correction, namely, programmed phase correction.

In programmed correction the method for introducing into the phase of the radiation a correcting predistortion calculated, for example, from the formula

$$\varphi_k(x,\ y) = -\frac{1}{2}\, k\, \varepsilon(z = 0,\ x,\ y) \int\limits_0^z \exp(-z'/z_s)\, dz',$$

where $k = 2\pi/\lambda$ is the wave number, is well known and widely employed.[2,6,8,9]

Figure 2 shows the dependence of γ on the effective thickness of the nonlinear layer. Given the predistortion φ_k makes it possible to achieve close to the minimum value of γ only up to $z_s \approx 0.7$–0.8, and already for $z_s \approx 1$ the angular divergence of the focused beam becomes greater than or comparable to the angular divergence of a collimated beam, i.e., for $z_s > 1$ such phase correction results in lower efficiency of radiation transfer.

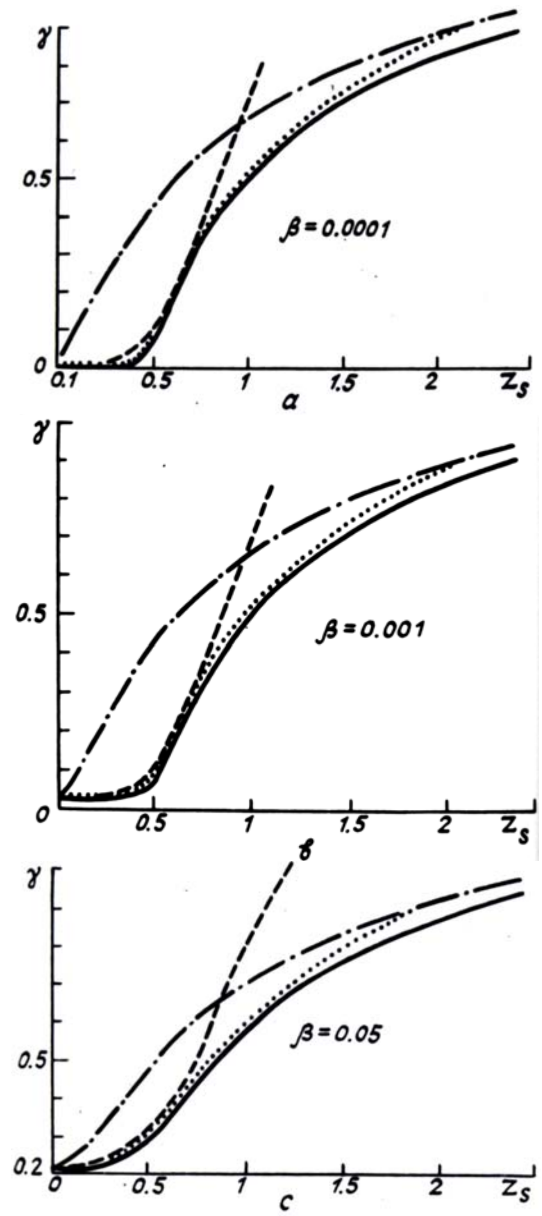

FIG. 2. The angular divergence γ versus the effective thickness of the nonlinear layer z_s for optimally focused (solid curve) and collimated (dot-dashed curve) beams as well as for a beam with predistortion φ_k (dashed curve) and a beam with phase correction determined by the programmed-phase-correction algorithm with a reference plane wave (dotted curve).

Thus the computational results based on the proposed scheme of adaptive correction can be compared with the calculations of the programmed phase correction (PPC) with predistortion φ_k. One can see that both methods make it possible to determine angular divergence close to the minimum only in a limited region of z_s. In addition, for $z_s = 4$ (for example, for $\beta = 0.0001$) the proposed adaptive correction gives a value of γ that is 23 times smaller than for a collimated beam; on the other hand, programmed correction with φ_k gives a value of γ that is only 6.5 times smaller than for a collimated beam.

Comparing the angular divergences of the collimated and optimally focused beams one can see that the efficiency of energy transfer cam also be Increased for $z_s > 1$. For this reason, we shall examine a different algorithm of programmed phase control, which, as the calculations showed, makes it possible to increase the efficiency of radiation transfer through a layer of a nonlinear medium with a thickness of up to two refraction lengths.

We studied above a scheme for adaptive control, the results of calculations based on which are presented in Fig. 1 and which permits finding the distribution of the phase front in the source plane for which the angular divergence either reaches a stationary level or its minimum value. This distribution of the phase front is introduced as a correcting predistortion for the transmitted beam. The numerical modeling of this scheme can be regarded as an algorithm for programmed phase correction.

The dashed curves in Fig. 2 show the calculation of the angular divergence based on such an algorithm. It follows from the calculations that for $z_s < 0.5$ an angular divergence of not more than 0.5 times greater them the minimum value is achieved. In addition, this excess also remains small for z_s of the order of the refraction length (≈ 1%). The excess reaches about 4–5% only for $z_s \approx 1.5$. The algorithm determines a close to optimal phase distribution in the initial plane within one to three iterations for $z_s \leq 0.4$ and $z_s > 0.8$; a large number of iterations is required only for $z_s \approx 0.5$ with $\beta \leq 0.01$.

We also note that the smaller the value of β the more efficient the radiation transfer to the point of reception is.

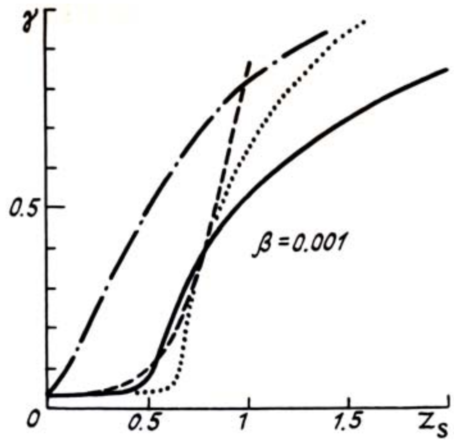

FIG. 3. The angular divergence γ as a function of z_s with nonlinearity of the wind type. f(z) is given by Eq. (6): collimated beam (dot-dashed curve), beam with predistortion φ_k (dashed curve), optimally focused beam (dotted curve); f(z) represented by Eq. (5): optimally focused beam (solid curve).

Figure 3 shows results that are analogous to Fig. 2b for the case when $f(z)$ is represented in the form

$$f(z) = \begin{cases} g^{-3}(z), & z \leq z_s, \\ 0, & z > z_s. \end{cases} \quad (6)$$

From a comparison of the results presented in these figures it follows that replacing the exponentially decreasing nonlinear refractive power in the layer by uniform refractive power with identical effective thickness of the layer results in appreciable errors.

Thus it can be concluded that in the case $\tau_{cor} \gg \tau_{nl}$ for the nonlinearity of the type (5) studied above programmed correction makes it possible to increase significantly (for $z_s = 0.4$ by a factor of 23 for $\beta = 0.0001$ and by a factor of 8 for $\beta = 0.001$) the efficiency of energy transfer as compared with the collimated beam. An algorithm for programmed phase control that permits calculating the angular divergence exceeding the minimum value by not more than 0.5–3% for $z_s \approx 1.5$ and by not more than 5% for z_s of the order of two refraction lengths was proposed. For lower values of β phase correction makes it possible to obtain high efficiency of radiation transfer.

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