THE EFFECT OF STIMULATED RAMAN SCATTERING ON THE PROPAGATION OF LASER BEAMS HAVING DIFFERENT PROFILES IN THE ATMOSPHERE

Yu.N. Kolomiets, S.S. Lebedev, and L.P. Semenov

Scientific–Production Union "Taifun," Obninsk

Received August 21, 1991

The problem of the stationary stimulated Raman scattering (SRS) is investigated by the methods of numerical simulation. A two–dimensional problem in the transverse coordinates is considered, which allows us to study the conditions of forming both spatially coherent and spatially incoherent Stokes beams. Laser beams having different spatial profiles are studied.

As is shown by theoretical estimates and experimental data, the phenomenon of stimulated Raman scattering (SRS) is of fundamental importance in the analysis of propagation of the intense laser radiation on the long paths in the atmosphere.[1,10]

The effect of the SRS on the beam characteristics was investigated by many authors (see, for example, Refs. 2 and 3), including the problem of propagation of the intense laser radiation in the atmosphere.[4,5] In the present paper the model of the stationary SRS without regard to the parametric interaction (which is applicable, e.g., for the rotational SRS of the circular polarized pumping radiation) is studied by the methods of numerical simulation. We consider the two–dimensional problem in the transverse coordinates which allows us to investigate the conditions of forming both the single–mode (spatially coherent) and multimode (spatially incoherent) Stokes beams. We determine the average characteristics of the scattered radiation by the method of numerical simulation. The initial laser beams having different profiles are studied.

Let the laser beam (the pump beam) propagating along the z axis be incident on the nonlinear medium. In the quasioptics approximation, neglecting the dispersion of the medium, the accompanying SRS is described by the following system of equations for the complex amplitudes of the laser– and Stokes– radiation components:

$$\left(\frac{\partial}{\partial z} + \frac{i}{2k_S}\Delta_\perp\right)A_S = \frac{1}{2}\, g\, A_l\, Q^* \,; \tag{1}$$

$$\left(\frac{\partial}{\partial z} + \frac{i}{2k_l}\Delta_\perp\right)A_l = -\frac{1}{2}\, g\, A_S\, \gamma\, Q \,; \tag{2}$$

$$\left(T_2\frac{\partial}{\partial \vartheta} + 1\right)Q = A_l A_S^* + N \,, \tag{3}$$

where A_l and A_S are the complex amplitudes of laser and Stokes radiation components normalized on the intensities $|A_l|^2 = I_l$ and $|A_S|^2 = I_S$; $\gamma = \omega_l/\omega_S$; ω_i, k_l, ω_S, and k_S are the frequencies and the wave vectors of the pumping and Stokes waves; $\vartheta = t - z/v_l$; $\nu = 1/v_l - 1/v_S$; v_l and v_S are the group velocities of pumping and Stokes waves; Q is the nondiagonal element of the density matrix; T_2 is the time of the transverse relaxation of the Raman transition; g is the amplification factor of the stationary SRS; $\Delta_\perp$ is the transverse Laplacian; N is the foreign source, characterizing the intrinsic noise of the medium, which can be considered as the Gaussian random process with the correlation function

$$\langle N(z, \mathbf{r}_\perp, t)\, N^*(z', \mathbf{r}'_\perp, t')\rangle =$$
$$= \left(\frac{2\pi}{k_S}\right)^2 \frac{4T_2}{\pi g^2}\left(\frac{ds}{d\Omega}\right)\delta(t - t')\,\delta(z - z')\,\delta(\mathbf{r}_\perp - \mathbf{r}'_\perp)\,, \tag{4}$$

where $\left(\frac{ds}{d\Omega}\right)$ is the spontaneous scattering cross section per unit volume of the medium and $\delta(x)$ is the Dirac delta function. Let us take

$$A_l(0, \mathbf{r}_\perp, t) = A_l^0(\mathbf{r}_\perp, t) \tag{5}$$

for the complex amplitude of the laser field.

As the boundary condition for the Stokes component we choose the zeroth–order oscillations of the vacuum field with the correlation function

$$\langle A_S(0, \mathbf{r}_\perp, t)\, A_S(0, \mathbf{r}'_\perp, t')\rangle = \chi\delta(t - t')\,\delta(\mathbf{r}_\perp - \mathbf{r}'_\perp)\,, \tag{6}$$

where $\chi = 8\pi\cdot\left(\frac{\hbar\omega_S}{2} + \hbar\omega_S\left[\exp\left(\frac{\hbar\omega_S}{2\pi k_B T}\right) - 1\right]^{-1}\right)$,

k_B is Boltzman's constant and T is the temperature of the medium. Equations (1)–(3) are valid in the absence of dispersion, when the pump intensity exceeds the critical value $I_l > I_{cr} = \Delta\omega\nu/g$, where $\Delta\omega$ is the width of the pump frequency spectrum.[6] We hereafter will consider the case in which the characteristic time of the change in the laser radiation intensity is much greater then the relaxation time T_2. In this case the stationary regime of the SRS is realized. The stationary regime of the SRS is realized on the rotational transitions of nitrogen molecules having the relaxation time $T_2 = 0.1$ ns at a pressure of ~ 1 atm for the pulses of duration > 10 ns. Let us reduce Eqs. (1)–(3) to the dimensionless form. Let us take

$$\mathbf{x}_{||} = z/(k_l a^2)\,,\ x_\perp = \mathbf{r}_\perp/a\,,\ \xi_l = A_l/E_l^0\,,\ \xi_S = A_S/E_l^0\,, \tag{7}$$

$$\tau = \theta/T^2\,,\ q = Q/(|E_l^0|^2 T^2)\,, \tag{8}$$

where a is the characteristic transverse size of the pump beam and E_l^0 is the characteristic value of the modulus of the complex pump amplitude. Then we obtain from Eqs. (1)–(3):

$$\frac{\partial}{\partial x_{||}}\xi_l + \frac{i}{2}\Delta_{x\perp}\xi_l = \gamma B\, \xi_S\, q\,; \tag{9}$$

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$$\frac{\partial}{\partial x_{||}}\xi_S + \frac{1}{\alpha}\frac{i}{2}\Delta_{x_\perp}\xi_S = -B\,\xi_l\,q^* ; \qquad (10)$$

$$\frac{\partial}{\partial\tau}q + q = \xi_l\,\xi_S^* + f . \qquad (11)$$

Here $B = \frac{1}{2}g\,I_0\,\kappa_l\,a^2$, $I_0 = |E_l^0|^2$ is the characteristic pump intensity, $a = \kappa_l/\kappa_S$ and $f = N/|E_l^0|^2$. The constant B has a sense of the amplification increment of the Stokes radiation over the diffraction length κa^2 of the laser beam.

On the starting section of the path the exhaustion of the pump beam owing to the SRS may be ignored. Therefore, the term on the right side of Eq. (9) may be omitted. Let us take the Fourier transforms of Eqs. (10) and (11) in τ and eliminate q from Eqs. (9)–(11). As a result, the system of Eqs. (9)–(11) takes the form

$$\frac{\partial}{\partial x_{||}}\xi_l + \frac{i}{2}\Delta_{x_\perp}\xi_l = 0 ; \qquad (12)$$

$$\frac{\partial}{\partial x_{||}}\xi_{S\beta} + \frac{1}{\alpha}\frac{i}{2}\Delta_{x_\perp}\xi_{S\beta} = \frac{B\,\alpha\,|\xi_l|^2}{1 + i\,\beta}\xi_{S\,\beta} + \frac{S}{1 + i\,\beta}\xi_l\,\tilde{n} , \qquad (13)$$

where $\xi_{S\beta} = 1/2\pi \int e^{i\,\beta\tau}\,\xi_S(x_{||},\,x_\perp,\,\tau)\,d\tau$ is the Fourier component of ξ_S, $\tilde{n}$ is the random source with the correlation function

$$\langle\tilde{n}\,(x_{||},\,\mathbf{x}_\perp,\,\beta)\,\tilde{n}^*(x_{||}',\,x_\perp',\,\beta')\rangle = \delta(x_{||} - x_{||}')\delta(\mathbf{x}_\perp - x_\perp')\delta(\beta - \beta') ,$$

and $S = (4\pi(d\sigma/d\Omega)/k_S{}^{1/2}$. Boundary condition (6) is reduced to the form

$$\langle\xi_{S\,\beta}(\mathbf{x}_\perp)\,x_{S\,\beta'}^*(\mathbf{x}_\perp')\rangle = \chi\,/(I\,_0T_2a^2)\;\delta(\beta - \beta')\;\delta(\mathbf{x}_\perp - x_\perp') .$$

Because the boundary condition for ξ_{Sb} and the source in Eq. (13) have Gaussian statistics, the solution of linear Eq. (13) is the Gaussian random function. If we are interested in the value of $\xi_S(x_{||},\,\mathbf{x}_\perp,\,\tau)$ at a certain fixed moment (for example, at $\tau = 0$), then obviously $\xi_S(x_{||},\,\mathbf{x}_\perp,\,0)$ will represent the random function with Gaussian statistics.

Let us reduce the term $\frac{|\xi_l|^2}{1 + i\,\beta}$ in Eq. (13) to the form $|\xi_l|^2 + |\xi_l^0|^2\left(\frac{1}{1 + i\,\beta} - 1\right)$, which approximates the initial function near $\beta \approx 0$ for all $\mathbf{x}_\perp$, and near the maximum of $|\xi_l|^2$ for all β, i.e., just in the range of variations of β and $\mathbf{x}_\perp$, where the most significant amplification of the Stokes wave occurs. Then the approximate relation for the function $\xi_S(x_{||},\,\mathbf{x}_\perp,\,0)$ can be represented as follows:

$$\xi_S(x_{||},\,x_\perp,\,0) = \left(\frac{S}{(2B)^{1/2}} + \frac{c}{E_0^2T_2a^2}\right)\times$$

$$\times\left(\frac{\pi}{2B\int_0^{x_{||}}|\xi_l^0|^2dx}\right)^{1/4}\tilde{\xi}_S(x_{||},\,x_\perp) , \qquad (14)$$

where $\xi_l^0 = \max\xi_l$ and $\tilde{\xi}_S$ is the solution of the equation

$$\frac{\partial}{\partial x_{||}}\tilde{\xi}_S + \frac{1}{\alpha}\frac{i}{2}\Delta_{x_\perp}\tilde{\xi}_S = B\,\alpha\,|\xi_l|^2\,\tilde{\xi}_S \qquad (15)$$

with the boundary condition in the form of the Gaussian random function, where

$$\langle\tilde{\xi}(x_\perp,\,0)\;\tilde{\xi}^*(\mathbf{x}_\perp',\,0)\rangle = \delta(\mathbf{x}_\perp - x_\perp') . \qquad (16)$$

Approximate relation (14) means that we neglect the effects involving "interweaving" of spatial and temporal variables, for example, with focusing of the Stokes radiation with the "lens" formed by the pump beam.

When $|\xi_S|$ becomes sufficiently large, the term describing the noise source in the right side of Eq. (11) may be omitted. In this case, eliminating q from Eqs. (9)–(11), we obtain

$$\frac{\partial}{\partial x_{||}}\xi_l + \frac{i}{2}\Delta_{x_\perp}\xi_l = -\gamma B\int_{-\infty}^{0}\xi_S(\tau)\,\xi_S^*(\tau + \tau')\,\xi_l(\tau + \tau')\,e^{\tau'}d\tau'; \quad (17)$$

$$\frac{\partial}{\partial x_{||}}\xi_S + \frac{i}{2\alpha}\Delta_{x_\perp}\xi_S = B\int_{-\infty}^{0}\xi_l(\tau)\xi_l^*(\tau + \tau')\,\xi_S(\tau + \tau')\,e^{\tau'}d\tau' . \quad (18)$$

On the starting section of the path, when we can neglect the exhaustion of pump, the lines of the Stokes radiation get narrower by a factor of $(M)^{1/2}$ (M is the total increment of amplification of the Stokes radiation).[6] Thus the functions ξ_S and ξ_l in Eqs. (17) and (18) in the first order can be considered to be slowly varying functions of time in comparison with e^s. In this approximation Eqs. (17) and (18) are reduced to the equations for the SRS–induced amplification of the monochromatic Stokes radiation in the regime of the pump beam exhaustion:

$$\frac{\partial}{\partial x_{||}}\xi_l + \frac{i}{2}\Delta_{x_\perp}\xi_l = -\gamma B|\xi_S|^2\,\xi_l ; \qquad (19)$$

$$\frac{\partial}{\partial x_{||}}\xi_S + \frac{i}{2\alpha}\Delta_{x_\perp}\xi_S = B\,|\xi_l|^2\,\xi_S . \qquad (20)$$

Accordingly, the solution of the problem on the SRS generation is reduced to the solution of Eqs. (12) and (15) on the starting section of the path, where the SRS is realized in the assigned pump field, and of Eqs. (19) and (20) in the regime of the pump beam exhaustion.

Since Eqs. (12) and (15) agree with Eqs. (19) and (20) except for the term in the right side of Eq. (19), which is unimportant on the starting section of the path, the solution of Eq. (14) yields the solution of the SRS problem not only on the starting section of the path but also in the regime of the pump beam exhaustion.

Equations (19) and (20) were solved numerically by the separation technique for physical factors.[7] We used the δ–correlated field given by Eq. (16) as a boundary condition for ξ_S at $x_{||} = 0$. The ensemble of realizations of the boundary conditions was averaged over for ξ_S. The calculations were made for the initial laser beams having Gaussian, super–Gaussian ($I \sim \exp(-(r/a)^n)$, $n = 4, 6, 8$) as well as annular profiles. The beams were chosen with identical maximum intensities and powers. The parameter B

varied from 50 to 300, which corresponded to the conditions of forming both spatially coherent (for small B) and the spatially incoherent (for large B) Stokes beams.

On the starting section of the path in the field of the laser radiation, the Stokes wave starts to form from the noise seed caused by the spontaneous radiation and by the vacuum fluctuations of the field at the boundary of the medium. Furthermore, if the path length exceeds the threshold value z_{th}, the energy is transferred from the pump beam into the Stokes beam. The calculated values of the dimensionless path length $x_{th} = z_{th}/\kappa a^2$ for the laser beams having different shapes are shown in Fig. 1 as a function of the parameter $B = 1/2gI_0 \kappa a^2$. The value of the longitudinal coordinate, for which ≈ 1 % of the laser beam energy was transferred into the Stokes beam, was taken as a threshold value.

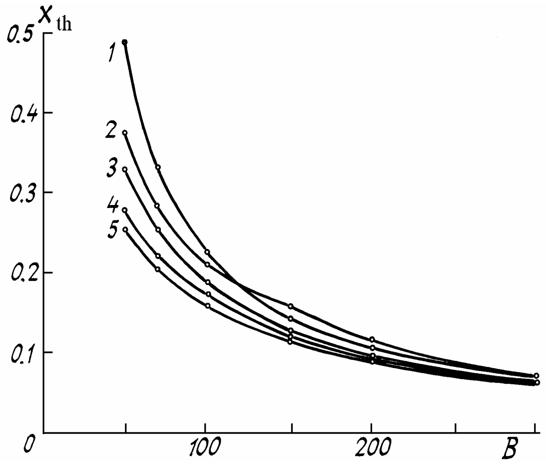

FIG. 1. The dependence of the dimensionless threshold path length $x_{th} = z_{th}/\kappa a^2$ on the parameter $B = 1/2\kappa a^2 gI$ for different shapes of the initial laser beams: 1) Gaussian, 2) annular, and 3 – 5) super–Gaussian (n = 4, 6, and 8 are shown by curves 3, 4, and 5).

In addition to the obvious fact that x_{th} decreases with increase of B one can easily see that for small B the value x_{th} strongly depends on the initial laser beam shape and this dependence becomes less pronounced when B increases. Such a behavior of x_{th} can be caused by a strong effect of the Fresnel diffraction of the laser beam for small B on the starting section of the path, and this effect decreases for large B. Moreover, such a behavior of the threshold value of the path length can be also associated with the dependence of the rate of increase of the increment of fundamental modes of the Stokes radiation on the shape of the incident radiation and on the parameter B.[8,9]

On the section of the path, where the longitudinal coordinate exceeded the threshold value x_{th}, the energy was transferred from the laser beam into the Stokes beam (the regime of pump beam exhaustion). Generally speaking, in this regime a certain increase of the Stokes beam divergence took place. This increase was mostly pronounced for large B. Therefore, for $B = 300$ the divergence increases approximately by a factor of 3. For small B this increase in the divergence becomes less pronounced in the regime of the pump beam exhaustion (for example, for $B = 50$ it practically does not occur). The same is true for the initial beams having other profiles. However, in these cases the divergence increase was less pronounced even for larger B.

Since the resulting Stokes beam has a large divergence, it starts to spread and its intensity decreases with increase of the longitudinal coordinate when the pump beam energy is transferred into it. The ratio of the Stokes beam intensity on the axis in the far–diffraction zone to the laser beam intensity in the far diffraction zone without the SRS as a function of the parameter B is shown in Fig. 2 for different profiles of the initial laser beam. It can be seen that for small B the strong dependence of the intensity of the transmitted Stokes beam on the profile of the incident beam takes place in the far–diffraction zone. When B increases, this dependence becomes less pronounced. Moreover, it can be seen from the figure, that the dependence of the intensity in the far diffraction zone on B (i.e., for the beam of a fixed radius on the incident beam intensity) has nonmonotonic character.

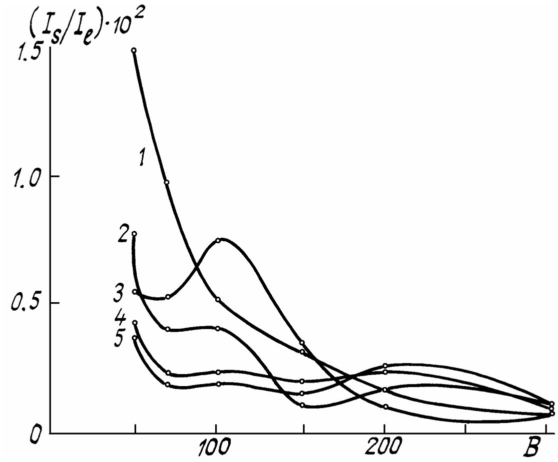

FIG. 2. The dependence of the ratio of the Stokes radiation intensity I_S in the presence of the SRS to the laser radiation intensity I_l without the SRS in the far–diffraction zone on the parameter B for different shapes of the initial laser beams: 1) Gaussian, 2) annular, and 3 – 5) super–Gaussian (n = 4, 6, and 8 are shown by curves 3, 4, and 5).

Apart from the average Stokes beam intensity, the variance of the intensity fluctuations, the correlation function, and the correlation radius were also determined in the calculation. The ratio of the Stokes beam radius r to its correlation radius r_c as a function of B are presented in Table I for the Gaussian, annular, and super–Gaussian (n = 4) initial laser beams.

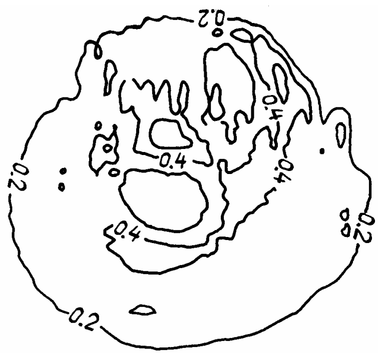

FIG. 3. The contour lines of the intensity of the Stokes beam for individual realization of the random seed for the Stokes component. The initial laser beam has a Gaussian profile for B = 300.

TABLE I.

B	r/r_c		
	Beams		
	Gaussian	Super–Gaussian	Annular
100	0.7	1.9	1.1
300	2.3	6.8	2.0
500	4.0	7.6	4.3

TABLE II.

B	$(\sigma_I)^{1/2}/I$		
	Beams		
	Gaussian	Super–Gaussian	Annular
100	0.14	0.04	0.05
300	0.35	0.75	0.92
500	0.57	0.75	0.80

The square of this ratio characterizes the number of the beam inhomogeneities. The results are given for the longitudinal coordinate corresponding to almost complete (≈ 80 %) energy transfer into the Stokes beam. When B increases, this ratio also increases, which testifies to the transition to the regime of formation of the multimode Stokes beam. This transition occurs for the super–Gaussian beams at smaller values of B than for the Gaussian and annular beams. The relative variance of the intensity fluctuations of the Stokes component on the beam axis for the longitudinal coordinate, being the same as in Table I, are presented in Table II. This ratio characterizes the degree of the amplitude modulation of the Stokes beam. It can be seen from Table I that the degree of the amplitude modulation increases with B. The obtained results testify the necessity of using the parabolic equations, two–dimensional in the transverse coordinates, for the description of the SRS for $B \geq 100$. The contour lines of the intensity of the Stokes radiation for individual realization of the random Stokes field, shown in Fig. 3 which can show the instantaneous intensity distribution at a certain random moment, are one more proof of this. The contour lines are given for the initial laser beam having the Gaussian shape for $B = 300$ and the value of the longitudinal coordinate corresponding to almost complete energy transfer into the Stokes beam. It can be seen that the solution is not axissymmetric.

The calculations allow us to draw the following conclusions:

1) The strong dependence of the average intensity of the transmitted beam on the shape of the incident beam is found to occur for a small incident beam power (the intensity varies by a factor of 3–4).

2) For large power this dependence is weak.

3) The number of the inhomogeneities of the transmitted beam depends on the incident beam shape for the entire range of variations of the incident beam power.

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