FORMATION OF LATERAL SHEAR INTERFEROGRAMS IN DIFFUSELY SCATTERED FIELDS USING DOUBLE-EXPOSURE RECORDINGS OF FOURIER HOLOGRAMS

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Analysis of a lateral shear interferometer based on the data of double–exposure recordings of a lens Fourier hologram of a matted screen is presented. It is shown both theoretically and experimentally that spatial filtering in the plane of the hologram makes it possible to control the quality of a lens or objective. The spatial filtration, if done in the far–diffraction zone, provides for recording the interference pattern characterizing the phase distortions introduced in the reference wave by the aberrations of the optical system forming it.

The method of double–exposure recordings of a lens Fourier hologram of a matted screen based on superposition of subjective speckle fields of two exposures in the plane of the photographic plate what yields the formation of lateral shear interference patterns is described in Ref. 1. Such interference patterns characterize the wave aberrations introduced by the lens or objective forming them as well as by aberrations of the quasiplanar wave front of radiation illuminating the matted screen. The same result is obtained in the case of a double–exposure recording of a lens Fourier hologram of a matted screen[2] by compensating the phase shift in the plane of the photographic plate introduced in the light wave by the lateral displacement of the matted screen by tilting the quasiplanar front of the reference wave.

In this paper we analyze the formation of lateral shear interferograms in the bands of infinite width in the case of double–exposure recordings of the lens Fourier holograms of a matted screen illuminated by aberrationless radiation of a diverging spherical wave.

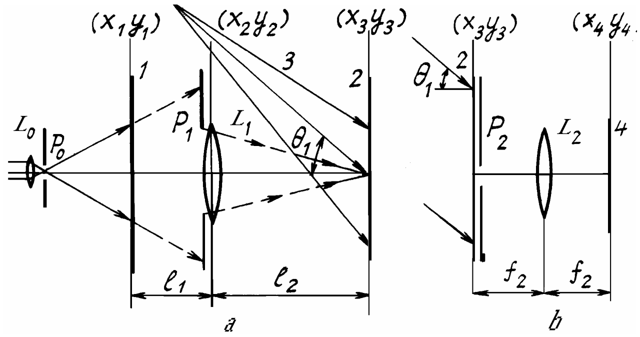


FIG. 1. Optical arrangements for recording (a) and reconstruction (b) of the double–exposure Fourier hologram of a matted screen: 1) matted screen, 2) photographic plate hologram, 3) reference beam, 4) and plane of recording the interference pattern. L_0, L_1, and L_2 are the lenses, p_0 and p_2 are the spatial filters, and p_1 is the aperture diaphragm.

As shown in Fig. 1*a*, the matted screen *1* is illuminated by an aberrationless diverging spherical wave having, in the plane of the screen, the radius of curvature R, the wave being formed with the lens L_0 and the point hole p_0 placed at its focus. During the first exposure the Fourier hologram of the matted screen is recorded on the photographic plate *2* with the lens L_1 and using a quarispherical diverging reference wave *3* having, in the plane of the photographic plate, the radius of curvature r. Before making the second exposure the matted screen is displaced in its plane, e.g., along the x axis by an amount a and the angle of incidence of a spatially limited reference beam is changed in the plane (x, r) from θ_1 to θ_2.

Let the complex amplitude of the field in the plane of the photographic plate be represented in the Fresnel approach and without account for constant amplitude and phase factors as

$$u_1(x_3, y_3) \sim \int\int_{-\infty}^{\infty}\int\int t(x_1, y_1) \exp\left[\frac{ik}{2R}(x_1^2 + y_1^2)\right] \times$$

$$\times \exp\left\{\frac{ik}{2l_1}[(x_1 - x_2)^2 + (y_1 - y_2)^2]\right\} p_1(x_2, y_2) \times$$

$$\times \exp\left\{- i\left[\frac{k}{2f_1}(x_2^2 + y_2^2) - \mathrm{j}_1(x_2, y_2)\right]\right\} \times$$

$$\times \exp\left\{\frac{ik}{2l_2}[(x_2 - x_3)^2 + (y_2 - y_3)^2]\right\} dx_1 dy_1 dx_2 dy_2 , \qquad (1)$$

where k is the wave number; $t(x_1, y_1)$ is the complex amplitude of the matted screen transparency, which is a random function of coordinates, $p_1(x_2, y_2) \exp i\varphi_1(x_2, y_2)$ is the generalized function of the pupil[3] of the lens L_1 (the focal length of the lens is f_1) which accounts for the axial wave aberrations, l_1 and l_2 are the distances from the principal section (x_2, y_2) of the lens L_1 to the matted screen and to the photographic plate, respectively.

If $\frac{1}{l_1} - \frac{1}{f_1} + \frac{1}{l_2} = \frac{1}{M} > 0$ and the condition $\frac{1}{R} + \frac{1}{l_1} - \frac{M}{l_1^2} = 0$ is fulfilled, relation (1) takes the form

$$u_1(x_3, y_3) \sim \exp\left[\frac{ik}{2l_2}(x_3^2 + y_3^2)\right]\left\{\exp\left[- \frac{ikM}{2l_2^2}(x_3^2 + y_3^2)\right] \times\right.$$

$$\times F\left[\frac{kMx_3}{l_1l_2}, \frac{kMy_3}{l_1l_2}\right] \otimes P_1(x_3, y_3)\Bigg\}, \qquad (2)$$

where symbol $\otimes$ denotes the operation of convolution;

$$F\left[\frac{kMx_3}{l_1l_2}, \frac{kMy_3}{l_1l_2}\right] = \int\int_{-\infty}^{\infty} t(x_1, y_1) \exp\left[-\frac{ikM}{l_1l_2}(x_1x_3 + y_1y_3)\right] dx_1 dy_1$$

is the Fourier transform of the matted screen transparency;

$$P_1(x_3, y_3) = \int\int_{-\infty}^{\infty} p_1(x_2, y_2) \exp i\varphi_1(x_2, y_2) \exp\left[-\frac{ik}{l_2}(x_2x_3 + y_2y_3)\right] \times$$

$\times\, dx_2 dy_2$ is the Fourier transform of the generalized function of the pupil of the lens L_1.

Since the width of the function $P_1(x_3, y_3)$ is of the order of $\frac{\lambda l_2}{d_1}$ (see Ref. 4), where λ is the wavelength of coherent light used for recording and reconstruction of the holograms and d_1 is the diameter of the lens L_1 pupil, we assume that within the interval of this function existence the phase shift of the spherical wave with the radius of curvature $\frac{l_2^2}{M}$ does not exceed π value. Then, for the region on the photographic plate with the diameter $D \le d_1 \frac{l_2}{M}$, one can remove the factor $\exp\left[-\frac{ikM}{2l_2^2}(x_3^2 + y_3^2)\right]$ characterizing the distribution of the spherical wave, from the integral of convolution integral (2)

$$u_1(x_3, y_3) \sim \exp\left[-\frac{ik(l_2 - M)}{2l_2^2}(x_3^2 + y_3^2)\right] \times$$

$$\times \left\{F\left[\frac{kMx_3}{l_1l_2}, \frac{kMy_3}{l_1l_2}\right] \otimes P_1(x_3, y_3)\right\}. \qquad (3)$$

If $l_1 > f_1$ relation (3) describes the Fourier transform of the input function being convoluted with the pulse amplitude response function of the lens L_1 multiplied by the quadratic phase term characterizing the distribution of the phase of a converging spherical wave. In this case the width of the pulse response exceeds that for the case of the exact Fourier transform[6] (i.e., when the matted screen is illuminated with a plane wave) because $l_2 > f_1$ and the transform scale changes with change of the matted screen position. The finite size of the lens L_1 restricts the length and lowers the upper frequency of the matted screen spatial spectrum with increasing l_1 that makes the range of the lens L_1 control over the field narrower. In the case of $l_1 < f_1$ the quadratic phase factor in Eq. (3) characterizes the distribution of the diverging spherical wave phase over the plane of the photographic plate. If the matted screen is superposed with the lens aperture, i.e., when $l_1 = 0$ the quadratic phase factor takes the form $\exp\left[\frac{ik}{2l_2^2}(x_3^2 + y_3^2)\right]$, the Fourier transform is scaled following the distance l_2, and no vignetting of the spatial spectrum of the matted screen by the lens L_1 takes place ($D = \infty$).

The distribution of the complex amplitude of a diffusely scattered field, corresponding to the second exposure, over the plane of the photographic plate takes, according to the known property of the Fourier transform, the form

$$u_2(x_3, y_3) \sim \exp\left[-\frac{ik(l_2 - M)}{2l_2^2}(x_3^2 + y_3^2)\right] \times$$

$$\times \left\{\exp\left(\frac{ikaMx_3}{l_1l_2}\right) F\left[\frac{kMx_3}{l_1l_2}, \frac{kMy_3}{l_1l_2}\right] \otimes P_1(x_3, y_3)\right\}. \qquad (4)$$

Let the complex amplitudes of the reference waves in the plane of the photographic plate be represented, in the approach used here, as follows

$$u_{01}(x_3, y_3) \sim \exp i\left[\frac{k}{2r}(x_3^2 + y_3^2) + kx_3\sin Q_1 + j_2(x_3, y_3)\right];$$

$$u_{02}(x_3, y_3) \sim \exp i\left[\frac{k}{2r}(x_3^2 + y_3^2) + kx_3\sin Q_2 + j_2(x_3 + b, y_3)\right];$$

where $r = \frac{l_2^2}{l_2 - M}$; $\varphi_2(x_3, y_3)$ is the deterministic function characterizing the phase distortions introduced into the reference wave by the wave aberrations of the optical system forming it; b is the value of shear due to the change of the dip angle of the spatially limited reference beam before the second exposure.

The distribution of intensity over the double-exposure hologram can be represented as the sum of intensities of the interferograms of the object and reference beams

$$I(x_3, y_3) \sim [u_1(x_3, y_3) + u_{01}(x_3, y_3)]\,[u_1(x_3, y_3) + u_{01}(x_3, y_3)]^* +$$

$$+ [u_2(x_3, y_3) + u_{02}(x_3, y_3)]\,[u_2(x_3, y_3) + u_{02}(x_3, y_3)]^*. \qquad (5)$$

Let us now assume that the dependence of the amplitude transmission of the hologram on the intensity is linear and let the hologram be transilluminated by a monochromatic quasiplanar wave propagating at the angle θ_1 with respect to the plane of the hologram and the complex amplitude be written as $\exp i\left[kx_3\sin\theta_1 + \varphi_3(x_3, y_3)\right]$, where $\varphi_3(x_3, y_3)$ is the deterministic function characterizing the phase distortions in the wave reconstructing the hologram owing to wave aberrations of the optical system forming it. Then the distribution of the field, in the minus first order of diffraction, over the plane of the hologram will take the form

$$u(x_3, y_3) \sim \exp i[-\varphi_2(x_3, y_3) + \varphi_3(x_3, y_3)] \left\{F\left[\frac{kMx_3}{l_1l_2}, \frac{kMy_3}{l_1l_2}\right] \otimes\right.$$

$$\left.\otimes P_1(x_3, y_3)\right\} + \exp i[kx_3\sin\Theta_1 - kx_3\sin\Theta_2 - \varphi_2(x_3 + b, y_3) +$$

$$+ \varphi_3(x_3, y_3)] \left\{\exp\left(\frac{ikaMx_3}{l_1l_2}\right) F\left[\frac{kMx_3}{l_1l_2}, \frac{kMy_3}{l_1l_2}\right] \otimes P_1(x_3, y_3)\right\}. \qquad (6)$$

If the condition $\sin\Theta_2 - \sin\Theta_1 + \frac{aM}{l_1l_2} = 0$ is satisfied Eq. (6) can be reduced to the form

$$u(x_3, y_3) \sim \exp i[-\varphi_2(x_3, y_3) + \varphi_3(x_3, y_3)] \left\{F\left[\frac{kMx_3}{l_1l_2}, \frac{kMy_3}{l_1l_2}\right] \otimes\right.$$

$$\left.\otimes P_1(x_3, y_3)\right\} + \exp i[-\varphi_2(x_3 + b, y_3) + \varphi_3(x_3, y_3)] \times$$

$$\times \left\{F\left[\frac{kMx_3}{l_1l_2}, \frac{kMy_3}{l_1l_2}\right] \otimes \exp\left(-\frac{ikaMx_3}{l_1l_2}\right) P_1(x_3, y_3)\right\}. \qquad (7)$$

As follows from Eq. (7) the subjective speckle fields of the two exposures coincide in the plane of the hologram at the angle $\alpha = \dfrac{aM}{l_1 l_2}$ between them and the information about the phase distortions introduced into the light wave by the lens L_1 is in individual speckle. As a consequence in the plane of the hologram we obtain the interference pattern produced due to the aberrations of the reference wave.[2] If an opaque screen with the round hole centered on the optical axis is placed in the plane of the hologram and if the condition $\varphi_2(x_3 + b, y_3) - \varphi_2(x_3, y_3) \le \pi$ is satisfied within the hole, i.e., the width of an interference fringe of the interference pattern localized on the hologram plane does not exceed the diameter of the filtering hole, then the diffraction field in the plane of filtration is described as follows:

$$u(x_3, y_3) \sim p_2(x_3, y_3) \exp i\varphi_3(x_3, y_3) \times$$
$$\times \left\{ F\left[\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right] \otimes \left[1 + \exp\left(-\frac{ikaMx_3}{l_1 l_2}\right)\right] P_1(x_3, y_3) \right\}, \quad (8)$$

where $p_2(x_3, y_3)$ is the transmission function of the screen with the round hole.[7]

Let the light field in the rear plane of the lens L_2 with the focal length f_2 (see Fig. 1) be represented by the Fourier integral of the light field in the plane of filtration. Then, based on the properties of the Fourier transform we have

$$u(x_4, y_4) \sim \Big\{ t(-\mu_1 x_4, -\mu_1 y_4)\, p_1(-\mu_2 x_4, -\mu_2 y_4) \exp i\varphi_1(-\mu_2 x_4, -\mu_2 y_4) +$$
$$+\ t(-\mu_2 x_4, -\mu_1 y_4)\, p_1\left(-\mu_2 x_4 - \frac{aM}{l_1}, -\mu_2 y_4\right) \times$$
$$\times \exp i\varphi_1\left(-\mu_2 x_4 - \frac{aM}{l_1}, -\mu_2 y_4\right)\Big\} \otimes P_2(x_4, y_4), \quad (9)$$

where $\mu_1 = \dfrac{l_1 l_2}{f_2 M}$ and $\mu_2 = \dfrac{l_2}{f_2}$ are the scaling factors;

$$P_2(x_4, y_4) = \int\!\!\int_{-\infty}^{\infty} p_2(x_3, y_3) \exp i\varphi_3(x_3, y_3) \exp\left[-\frac{ik}{f_2}(x_3 x_4 + y_3 y_4)\right] \mathrm{d}x_3 \mathrm{d}y_3$$

is the Fourier transform of the filtering screen transmission function with an account of the phase distortions of the wave reconstructing the hologram or for deviations of the photographic plate surface from the flat one without such aberrations.

It follows from Eq. (9) that in the plane of the matted screen image, within the region of overlapping of the images of the pupil of the lens L_1, the identical speckles coincide and, hence, the interference pattern is localized in the plane (x_4, y_4) and, as a result of the conditions formulated in Refs. 1 and 2, the superposition of correlated speckle fields of the two records gives the following distribution of the light intensity over the recording plane 4 (see Fig. 1*b*):

$$I(x_4, y_4) \sim \left\{1 + \cos\left[\varphi_1(-\mu_2 x_4, -\mu_2 y_4) - \right.\right.$$
$$\left.\left. - \varphi_1\left(-\mu_2 x_4 - \frac{aM}{l_1}, -\mu_2 y_4\right)\right]\right\} \left| t(-\mu_1 x_4, -\mu_1 y_4) \otimes P_2(x_4, y_4)\right|^2. \quad (10)$$

This distribution describes the speckle structure modulated by the interference fringes. The interference pattern has the form of the lateral shear interferogram in the bands of infinite width, which characterizes the axial wave aberrations of the lens L_1. Displacement of the filtering hole center towards the axis of the matted screen makes it possible to control the lens L_1 over its field[8] as was demonstrated experimentally in Ref. 9. It should be noted that in order to obtain the interference pattern within the entire pupil of the lens L_1 it is necessary, as it follows from Eq. (10), that the diameter d_p of the illuminated region of the matted screen exceeds the value $d_p \ge d_1 \dfrac{M}{l_1}$.

Let us now assume that the diameter of the lens L_2 (see Fig. 2) exceeds the size of the hologram and the lens L_2 itself is in the plane of the hologram. In this case the field of diffraction in the focal plane of this lens is

$$u(x_4, y_4) \sim \exp\left[\frac{ik}{2f_2}(x_4^2 + y_4^2)\right] \Big\{ \Phi_0(x_4, y_4) \otimes t(-\mu_1 x_4, -\mu_1 y_4) \times$$
$$\times\ p_1(-\mu_2 x_4, -\mu_2 y_4) \exp i\varphi_1(-\mu_2 x_4, -\mu_2 y_4) + \Phi_1(x_4, y_4) \otimes$$
$$\otimes\, t(-\mu_1 x_4, -\mu_1 y_4)\, p_1\left(-\mu_2 x_4, -\frac{aM}{l_1} - \mu_2 y_4\right) \times$$
$$\times \exp i\varphi_1\left(-\mu_2 x_4 - \frac{aM}{l_1}, -\mu_2 y_4\right)\Big\}, \quad (11)$$

where

$$\Phi_0(x_4, y_4) = \int\!\!\int_{-\infty}^{\infty} \exp i[\varphi_3(x_3, y_3) - \varphi_2(x_3, y_3)] \times$$
$$\times \exp\left[-\frac{ik}{f_2}(x_3 x_4 + y_3 y_4)\right] \mathrm{d}x_3 \mathrm{d}y_3 \text{ and}$$

$$\Phi_1(x_4, y_4) = \int\!\!\int_{-\infty}^{\infty} \exp i[\varphi_3(x_3, y_3) - \varphi_2(x_3 + b, y_3)] \times$$
$$\times \exp\left[-\frac{ik}{f_2}(x_3 x_4 + y_3 y_4)\right] \mathrm{d}x_3 \mathrm{d}y_3$$

are the Fourier transforms of the corresponding functions.

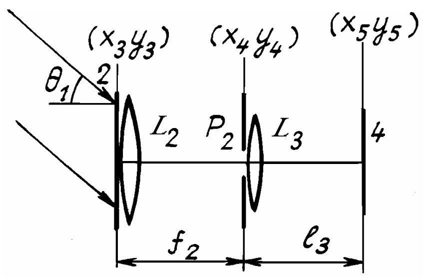


FIG. 2. Optical arrangement of recording the interference pattern localized in the plane of the hologram.

If, additionaly, an opaque screen with the round hole centered at the optical axis of the lens L_2 is placed in its focal plane and the diameter of the hole does not exceed the width of an interference band of the interference pattern localized in the far–diffraction zone, i.e.,

$$\varphi_1(-\mu_2 x_4, -\mu_2 y_4) - \varphi_1\left(-\mu_2 x_4 - \frac{aM}{l_1}, -\mu_2 y_4\right) \le \pi$$

then, at the exit of the filtering screen, we obtain the diffraction field

$$u(x_4, y_4) \sim p_2(x_4, y_4) \exp\left[\frac{ik}{2f_2}(x_4^2 + y_4^2)\right]\{\Phi_0(x_4, y_4) \otimes$$

$$\otimes\, t(-\mu_1 x_4, -\mu_1 y_4) + \Phi_1(x_4, y_4) \otimes t(-\mu_1 x_4, -\mu_1 y_4)\} . \quad (12)$$

Let the spatial filtering be performed using the aperture diaphragm of the lens L_3 (see Fig. 2) having the focal length f_3. It is also assumed that the condition $\frac{1}{f_2} - \frac{1}{f_3} + \frac{1}{l} = 0$, where l is the distance between the planes (x_4, y_4) and (x_5, y_5), is satisfied. Then the light field in the recording plane *4* can be written as follows:

$$u(x_5, y_5) \sim \exp\left[\frac{ik}{2l}(x_5^2 + y_5^2)\right] \int_{-\infty}^{\infty}\!\!\int u(x_4, y_4) \times$$

$$\times \exp\left[-\frac{ik}{2f_2}(x_4^2 + y_4^2)\right] \exp\left[-\frac{ik}{l}(x_4 x_5 + y_4 y_5)\right] dx_4 dy_4 . \quad (13)$$

By substituting expression (12) in relation (13) we have

$$u(x_5, y_5) \sim \exp\left[\frac{ik}{2l}(x_5^2 + y_5^2)\right] \{\exp i[\varphi_3(-\mu_3 x_5, -\mu_3 y_5) -$$

$$- \varphi_2(-\mu_3 x_5, -\mu_3 y_5)] F[t(-\mu_1 x_4, -\mu_1 y_4)] + \exp i[\varphi_3(-\mu_3 x_5, -\mu_3 y_5) -$$

$$- \varphi_2(-\mu_3 x_5 + b, -\mu_3 y_5)] F[t(-\mu_1 x_4, -\mu_1 y_4)]\} \otimes P_2(x_5, y_5)\}, (14)$$

where $\mu_3 = \frac{f_2}{l}$ is the scaling factor

$$F[t(-\mu_1 x_4, \mu_1 y_4)] = \int_{-\infty}^{\infty}\!\!\int t(-\mu_1 x_4, -\mu_1 y_4) \times$$

$$\times \exp\left[-\frac{ik}{l}(x_4 x_5 + y_4 y_5)\right] dx_4 dy, \text{ and}$$

$$P_2(x_5, y_5) = \int_{-\infty}^{\infty}\!\!\int p_2(x_4, y_4) \exp\left[-\frac{ik}{l}(x_4 x_5 + y_4 y_5)\right] dx_4 dy$$

are the Fourier transforms of the corresponding functions.

As it follows from relation (14) the speckle fields recorded at two exposures are superposed and identical speckles in them coincide. Since the width of the function $P_2(x_5, y_5)$ determines the size of a subject speckle in the plane (x_5, y_5) one can, assuming the period of the function $\exp i\varphi_2(-\mu_3 x_5, -\mu_3 y_5) + \exp i\varphi_2(-\mu_3 x_5 + b, -\mu_3 y_5)$ to be at least an order of magnitude larger than the size of a speckle,[10] remove this function from the integrand of the convolution integral. As a result the irradiance distribution over the plane (x_5, y_5) can be given in the form

$$I(x_5, y_5) \sim \{1 + \cos[\varphi_2(-\mu_3 x_5 + b, -\mu_3 y_5) - \varphi_2(-\mu_3 x_5, -\mu_3 y_5)\} \times$$

$$\times \left| F[t(-\mu_1 x_4, -\mu_1 y_4)] \exp i\varphi_3(-\mu_3 x_5, -\mu_3 y_5) \otimes P_2(x_5, y_5) \right|^2 . (15)$$

This equation desribes the speckle structure modulated by the interference fringes. The interference pattern, in this case, has the view of a lateral shear interferogram in the interference bands of infinite width characterizing the phase distortions of a spatially limited reference beam introduced by the wave aberrations of the optical system forming it.

As well known,[11] to observe the interference pattern in a real time scale the interference patterns of the object and reference waves are to be recorded on the photographic plate. Then the obtained hologram is placed exactly at the position of the photographic plate during its exposure and the hologram is illuminated by the initial reference and objective waves. Keeping this in mind let us consider the case of reconstructing the double–exposure Fourier hologram with a copy of the reference wave corresponding to the second exposure. In this case the distribution of the diffraction field over the plane of the hologram in the minus first order of diffraction takes the form

$$u(x_3, y_3) \sim \exp\left[\frac{ik}{2r}(x_3^2 + y_3^2)\right] \exp i\varphi_3(x_3, y_3)\left\{\exp i[\varphi_2(x_3 + b, y_3) -\right.$$

$$- \varphi_2(x_3, y_3)] \left\{F\left(\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right) \otimes P_1(x_3, y_3)\right\} +$$

$$\left. + F\left(\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right) \otimes \exp\left(-\frac{ikaMx_3}{l_1 l_2}\right) P_1(x_3, y_3)\right\} . \quad (16)$$

If one makes the spatial filtering of the diffraction field on the optical axis in the plane of the hologram using the aperture diaphragm p_2 of the lens L_2 (see Fig. 3*a*), then, at the distance l from it, which satisfies the condition $\frac{1}{r} - \frac{1}{f_2} + \frac{1}{l} = 0$, the light field is described by the relation

$$u(x_4, y_4) \sim \exp\left[\frac{ik}{2l}(x_4^2 + y_4^2)\right] \left\{t(-\mu_1' x_4, -\mu_1' y_4) \times\right.$$

$$\times p_1(-\mu_2' x_4, -\mu_2' y_4) \exp i\varphi_1(-\mu_2' x_4, -\mu_2' y_4) \otimes P_2'(x_4, y_4) +$$

$$+ t(-\mu_1' x_4, -\mu_1' y_4)\, p_1\left(-\mu_2' x_4 - \frac{aM}{l_1}, -\mu_2' y_4\right) \times$$

$$\left. \times \exp i\varphi_1\left(-\mu_2' x_4 - \frac{aM}{l_1}, -\mu_2' y_4\right) \otimes P_2'(x_4, y_4)\right\} , \quad (17)$$

where $\mu_1' = \frac{l_1 l_2}{lM}$ and $\mu_2' = \frac{l_2}{l}$ are the scaling factors and

$$P_2'(x_4, y_4) =$$

$$= \int_{-\infty}^{\infty}\!\!\int p_2(x_3, y_3) \exp i\varphi_3(x_3, y_3) \exp\left[-\frac{ik}{l}(x_3 x_4 + y_3 y_4)\right] dx_3 dy_3$$

is the Fourier transform of the transmission function of the filtering screen with an account of deviations of the photographic plate surface from the perfectly flat.

The superposition of the correlated speckle–fields (17) yields the distribution of the irradiance over the plane of recording (x_4, y_4)

$$I(x_4, y_4) \sim \left\{1 + \cos\left[\varphi_1(-\mu_2' x_4, -\mu_2' y_4) - \varphi_1\left(-\mu_2' x_4 - \frac{aM}{l_1}, -\mu_2' y_4\right)\right]\right\} \left| t(-\mu_1' x_4, -\mu_1' y_4) \otimes P_2'(x_4, y_4)\right|^2. \quad (18)$$

A comparison of relations (10) and (18) shows that in the case of reconstruction of a double–exposure Fourier hologram with a copy of the reference wave used in the second exposure the characteristic size of a speckle in the plane of recording of the interference pattern increases since $l > f_2$. However, a simultaneously observed increase of the interference pattern scale leads to the situation when the sensitivity of the lateral shear interferometer, using the diffusely scattered fields to the wave aberrations of the lens under control remains constant.

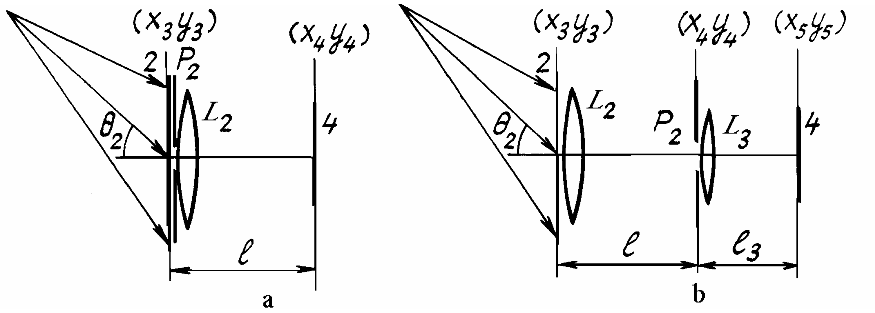
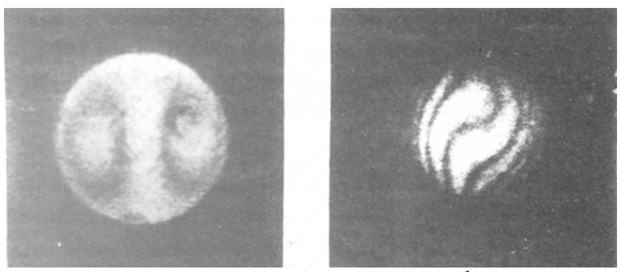


FIG. 3. Optical arrangement of recording the interference pattern when reconstructing the double–exposure Fourier hologram with the coherent radiation in the form of diverging spherical wave and using spatial filtration in the plane of the hologram (a) and in the plane of the image of the matted screen (b).

It can be shown that in the considered case of reconstructing the double–exposure Fourier hologram with the help of a spherical wave the recording of the interference pattern localized in the plane of the hologram in the minus first order of diffraction can be performed by spatial filtering on the optical axis in the plane (x_4, y_4) (see Fig. 3*b*) using the aperture diaphragm of the lens L_3 provided that the condition $\frac{1}{l} - \frac{1}{f_3} + \frac{1}{l_3} = 0$ is satisfied, where f_3 is the focal length of the lens L_3 and l_3 is the distance from its principal plane (x_4, y_4) to the plane (x_5, y_5).

It should be noted that if the double–exposure hologram is recorded according to Fig. 1*a* with the use of a quasiplanar reference wave, while its reconstruction is being performed using a copy of the reference wave corresponding, for instance, to that used in the second exposure, then the distribution of field over the plane of the hologram in the minus first order of diffraction is described by Eq. (16). As a consequence, the recording of the interference pattern characterizing the wave aberrations of the lens L_1 can be made according to Fig. 3*a*, while the recording of the interference pattern localized in the plane of the hologram–according to Fig. 3*b*. It should be taken into account that in this case the reconstruction of the double–exposure hologram is performed using a quasiplanar wave. However in this case the width of the spatial frequency spectrum of the hologram is always greater than that for the Fourier hologram,[12] other parameters of the holographing arrangement being the same as in the Fourier holographing scheme. For this reason, at a fixed spatial resolution of the medium used to record the hologram, the range of the lens or objective control over the field decreases.

In our experiments we used the Micrat VRL–type photographic plates to record the double–exposure Fourier holograms of a matted screen and the He–Ne laser radiation at $\lambda = 0.63$ μm to illuminate the screen. Figure 4 presents an example of the interferogram recorded using the spatial filtering on the optical axis in the plane of the hologram and reconstructed using a narrow (2 mm in diameter) laser beam. The interference pattern characterizes the spherical aberration of the lens with the focal length $f_1 = 130$ mm, and the pupil diameter $d_1 = 25$ mm, and postfocal defocusing. Using this lens we have recorded double–exposure Fourier holograms of a matted screen for $l_1 = 80$ mm, $l_1 = 200$ mm and $R = 291$ mm. In the optical channel forming the reference wave the laser beam was expanded and then using a converging lens, we formed a diverging reference beam with the radius of curvature of the wave front at the plane of the photographic plate $r = 408$ mm. Before making the record of the second exposure the matted screen was displaced along the direction perpendicular to the optical axis by the amount $a = 0.4 \pm 0.002$ mm and the angle of incidence of the reference beam was changed by $\Delta\theta = 8'40'' + 10''$.



a *b*

FIG. 4. The lateral shear interferograms recorded with the spatial filtering on the optical axis in the plane of the hologram (a) and in the plane of the image of a matted screen (b).

Figure 4*b* presents the interferogram, recorded using the spatial filtration on the optical axis in the plane of the matted screen image, that characterizes the phase distortions introduced into the reference wave by the aberrations of the optical system forming it. In this case the double–exposure hologram was reconstructed using a collimated beam 100 mm in diameter and a collimating system composed of an objective 500 mm in focal length and the pupil 100 mm in diameter and an eyepiece 80 mm in focal length and the pupil 20 mm in diameter was used for constructing the real image of the plane of the hologram. The filtering hole placed in the frequency plane has a diameter of 3 mm. The length of the interferogram presented in Fig. 4*b* was limited due to the vignetting of the diffusely scattered field by the lens under control and was equal to 50 ± 1 mm, that well agrees with the calculated value.

In conclusion it should be noted that the above–discussed method of the double–exposure recording of the lens Fourier hologram of a matted screen without spatial filtering in the channel of formation of a diverging reference beam yields the formation of the lateral shear interference patterns in the bands of infinite width. In this case the interference pattern characterizing the aberrations of the reference beam is localized in the plane of the hologram while that characterizing the aberrations of the lens is in the far–diffraction zone. It is possible to observe them separately by spatially filtering the diffraction of a diffusely scattered field in the relevant planes. In addition, at a fixed spatial resolution of the medium used to record the hologram, this method enables one to widen the range of controlling a lens over the field owing to the increase of the bandwidth of the spatial frequencies of the matted screen transmitted through the lens under control.

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