The calculation of strong electric field in an open discharge

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The method is described for determining parameters of the cathode drop region based on simultaneous solution of the discharge equations and an equation describing the development of the avalanche of running away electrons. The paper presents the calculated results as well as simple relationships between the parameters. A possibility for a high-voltage medium-pressure gas discharge to exist in a quasi–stationary mode is shown and the conditions for this are determined. Based on the data obtained a mechanism of and conditions for the photoelectronic discharge to occur is discussed.

A pulsed discharge in a narrow gap between the parallel solid cathode and grid anode, followed by an extended gas–filled region, is called the open discharge. It is an efficient source of an electron beam (EB) with the power from 1 to 10 keV.[1,2] No efficient theory of the open discharge has been developed so far. The problem is in the complexity of the description of gas ionization by electrons, most of those are running away electrons because their velocity (energy) distribution is local and depends on the history of each electron. The methods developed for this purpose of solving the Boltzmann kinetic equation, based on the Monte Carlo methods,[3,5] are very cumbersome and calculated results are too complicated for analysis. When the electric field distribution is unknown while being a required characteristic, the above mentioned methods turned out inefficient. The pulsed discharge with running away electrons is just this case.

In the Ref. 6 to describe the running away electron avalanche, the ionization density function $w(x) = \mathrm{d}K(x)/\mathrm{d}x$ was introduced, instead of the first Townsend coefficient, where $K(x)$ is the number of running away electrons at a distance x from the start of the first electron. The function $w(x)$ is defined by the following equation:

$$w(x) = \vartheta\, n \left\{ \sigma_i(0, x) + \int\limits_0^x w(\xi)\, \sigma_i(\xi, x)\, \mathrm{d}\xi \right\}, \quad (1)$$

where n is the gas density; $\sigma_i(\xi, x)$ is the cross section of gas particles ionization at a point x by a cascade electron produced at a point ξ; ϑ is the coefficient of electron path extension due to scattering. The quantity ϑ equals 1.3 to 1.4 in the fields exceeding twice the running away threshold and rapidly approaches 1 with the field strengthening.[7] Similar function is used[8] when investigating the initial phase of the breakdown at low gas pressures. In the present paper this approach is applied for the first time to calculate the parameters of the strong field region in the pulsed gas discharge at a medium gas pressure. This paper describes the solution of these problems and their analysis.

1. The calculation technique

The problem is solved in the quasi-stationary approximation. The anode is transparent for the beam electrons, therefore their back reflection to the discharge gap is neglected. The volume electron charge in the strong field is also ignored. In the first approximation we can drop the gas ionization by fast ions and neutral particles. The gas is helium. We consider the following set of equations:

– the equation of charge generation in a gas (1). In short interelectrode gaps the magnitude of the coefficient even in the relatively weak fields can be taken to be unity[7];

– the Dravin formula with the unit fitting coefficients for the gas ionization cross section by electron impact. For helium the cross section equals

$$\sigma_i(z) = \sigma_0\, g(z), \quad \sigma_0 = 1.43\cdot 10^{-20}\ \mathrm{m}^2,$$
$$g(z) = [(z-1)/z^2]\ \ln(5z/4), \quad z = \varepsilon/J, \quad (2)$$

where ε is the electron energy, J is the ionization potential of a helium atom;

– the equation of current continuity

$$\partial j_+(x)/\partial x = -\, j_{e0}\, w(x), \quad (3)$$

where j_+ and j_{e0} are the ion current density in the interelectrode gap and the electron current density at the cathode;

– the Poisson equation

$$\partial E(x)/\partial x = -\,[j_+(x)/\varepsilon_0\, v_+(x)], \quad (4)$$

where ε_0 is the dielectric constant, $v_+(x)$ is the ion drift velocity;

– the law of ion motion in gas

$$v_+(x) = \Gamma \sqrt{E(x)/n}, \quad \Gamma = 1 \cdot 10^{13}; \quad (5)$$

– the dependence of the coefficient of the electron cathode emission γ on the field intensity at the cathode $E(0)$. According to the measurement data[9] this dependence can be approximated by the following analytical expression:

$$\gamma = 3\cdot 10^{-4}\ [e\,(0)/p\,] - 0.44, \quad (6)$$

where $[E/P]$ is measured in V/(Pa · m). This expression corresponds to the experimental data

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accurate to the experimental errors at the energy of ions and fast atoms in the range from 100 to 1000 eV.

Traditionally, the set of discharge equations is complemented by the normalization equation and the boundary conditions[10] and is solved by numerical methods.

The boundary conditions at the cathode and anode, if any, make the solution of equations more difficult. Therefore in this case another approach is used. The data on the cathode necessary for making the calculations are set beforehand, and after the simulation run the values of all the remaining parameters are obtained. As the free parameters the electron current density j_{e0} and the field strength $E(0)$ at the cathode were left.

Under conditions favorable for running away the cross section $\sigma_i(\xi, x)$ in Eq. (1) depends only on the potential difference an electron is at.[6] Therefore the transition to an independent variable $z = -e\,\varphi(x)/J$ (where $\varphi(x)$ is the field potential at a point x) enables one to reduce the set of equations (1)–(5) to the following equation:

$$Y''(z) = \frac{R}{Y^{0.4}(z)}\left\{G\,g(z) + \int\limits_1^{z-1} g(z-\zeta)\,Y''(\zeta)\,\mathrm{d}\zeta\right\}, \quad (7)$$

where

$$Y(z) = \left(\frac{E(z)}{E(0)}\right)^{5/2}; \; R = \frac{\sigma_0\,J\,n}{e\,E(0)}\,; \; G = \frac{2.5\,J\,j_{e0}\,n^{1/2}}{e\,\varepsilon_0\,\Gamma\,E^{5/2}(0)} \quad (8)$$

with the initial conditions

$$Y(0) = 1, \quad Y'(0) = -\,G/\gamma, \quad Y''(0) = 0. \quad (9)$$

In terms of the function $Y(z)$ the parameters of interest are determined as follows:

$$U = J\,z; \quad x = \frac{J}{E(0)}\int\limits_0^z \frac{\mathrm{d}\zeta}{Y^{0.4}(\zeta)}\,; \quad \frac{j_+(z)}{j_+(0)} = \frac{Y'(z)}{Y'(0)}\,;$$

$$K(z) = \frac{1}{\gamma}\left[1 - \frac{j_+(z)}{j_+(0)}\right]; \; n_+(z) = 0.9\cdot 10^6\,\frac{E^2(0)\;Y'(z)}{Y^{0.2}(z)}\,. \quad (10)$$

Here n_+ is the ion concentration; $K(z)$ is the factor of charge multiplication in a gas.

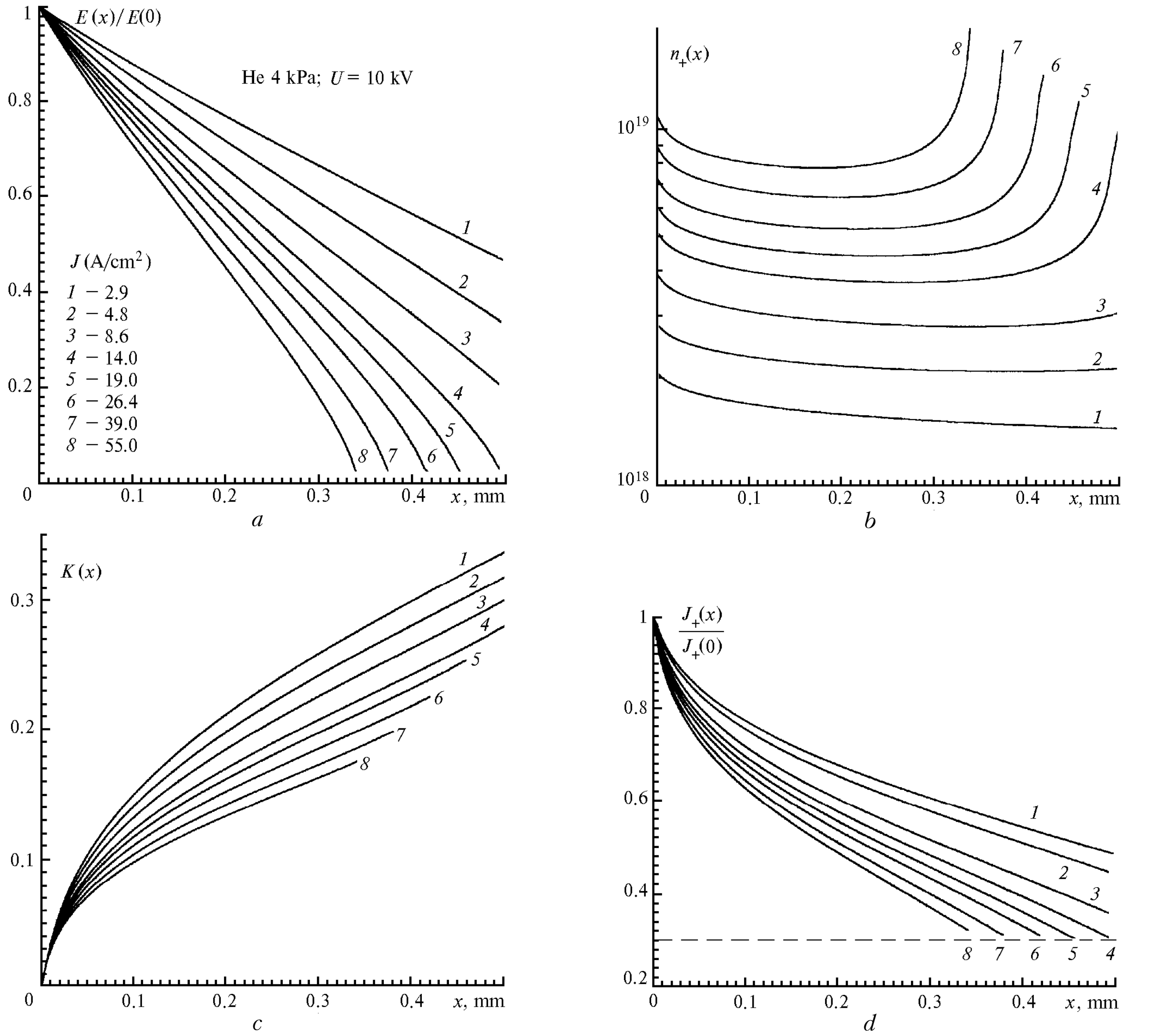

Fig. 1. Distribution of electric field intensity (a), ion density (b), the charge multiplication factor K(c), and the ion current density (d) over the interelectrode gap length at separate values of j_{e0}.

2. General analysis of solutions

Now we consider a typical problem with the following parameters: d is the electrode gap, U_0 is the voltage applied to it, and P is the gas pressure. Figure 1 shows solutions to this problem at different values of j_{e0}. At a low current the region of strong field occupies the entire electrode gap (curves *1–4*) and, therefore, the set of equations (1)–(6) describes the discharge as a whole. As the current increases the region of strong field is being localized within the region of the length l_k close to the cathode (curves *5–8*) and the cathode drop (CD) occurs there while the plasma develops in the remaining part of the gap. The nature of the strength $E(x)$ variations is rather complicated. In the long electrode gaps and low discharge current the gradient $\mathrm{d}E(x)/\mathrm{d}x$ increases smoothly up to zero, and, on the contrary, at a high current it decreases when approaching the plasma boundary. The calculated results reveal the first property characteristic of the discharge that the current amplification in the strong field region for all the solutions shown in Fig. 1 turned out to be insufficient for a self-maintained discharge to occur: $\gamma K(d)$ (curves *1–4*), $\gamma K(I_k)$ (curves *5–8*) < 1. By this we mean that in the former case the discharge occurs with ion current from the region beyond the anode and in the latter one the source of this current is the near-anode plasma. Therefore the criterion of discharge stationarity has the form:

$$\gamma K\{d, l_k\} = 1 - r, \quad r = j_+\{d, l_k\}/j_+(0). \qquad (11)$$

Thus, the cathode drop region (CDR) in this discharge is not self-sufficient in contrast to the anomalous glow discharge.[10–12] It turned out that in the CDR formed ($I_k < d$), the quantity r is independent of the discharge current, and its dependence on voltage can be approximated by the expression

$$r \approx 0.272 \ln [32/U_0], \qquad (12)$$

where U_0 is in kilovolts. As the discharge current j_p changes, the CDR dimensions vary following the law $l_k \sim j_p^{-m}$, and the field strength at the cathode – $E(0) \sim j_p^m$, where $m = 0.3$ in the range of U_0 variation from 4 to 10 kV. All the above mentioned relationships were obtained at a constant helium pressure of 4 kPa and the linear dependence of the coefficient γ on E/P according to Eq. (6). As the function (6) changes, the ratios between the parameters of the CDR will be different and r depends on the discharge current.

In the general case, to describe the discharge at $l_k < d$, the set of equations (1)–(6) should be supplemented by the relationships determining the generation of charges and their flows in plasma in the discharge gap segment $d - l_k$. Note that not the entire $(d - l_k)$ segment, from the CDR boundary to the anode, contributes to formation of the ion current $j_+(l_k)$, but only its part of the thickness l_{ar}. The calculations allowing for the volume electron charge in the Poisson equation indicates that at the CDR and plasma boundary the field strength is low. There is a thin layer of plasma, where the degree of gas ionization is about one order of magnitude higher than that in other parts of the discharge gap.[12] In this layer the ionization can be produced by the electrons injected from the CDR. The charge multiplication factor in the layer equals

$$K_{ar} = w(l_k)\, l_{ar},$$

and then

$$r = \gamma\, K_{ar}. \qquad (13)$$

Next, having in mind the expression $w(l_k) = [\mathrm{d}K_{CDR}(l_k)/\mathrm{d}l_k]$ and assuming that $l_{ar} \approx d - l_k$, we derive from Eq. (11) the following equation connecting both of the discharge regions:

$$K_{CDR}(l_k) = \frac{1}{\gamma} - (d - l_k) \left.\frac{\mathrm{d}K_{CDR}(x)}{\mathrm{d}x}\right|_{x = l_k}. \qquad (14)$$

To solve this equation, let us write, based on Eq. (6), the coefficient γ in the form

$$\gamma = \theta + \rho/l_k \qquad (15)$$

and introduce the parameters $\beta = l_k/d$, $S = -\rho/(\theta d)$. Then the solution of Eq. (15) takes the form

$$K_{CDR} = K(\beta) = -\frac{1}{\theta}\frac{1}{S - 1} \times$$

$$\times \left\{ \frac{S}{S - 1}(1 - \beta) \ln \left| \frac{1 - \beta}{1 - \beta/S} \right| + \beta \right\}. \qquad (16)$$

Because $K(\beta)$ and r are determined by the set of equations (1)–(5) and (15), the relation (16) gives a specific value β at which all the conditions are fulfilled. Thus, from the set of solutions (Fig. 1) only one solution remains, which is valid at specified values of U_0, d, n (or P). The calculations have also shown that stationary solutions can be obtained at β from 0.95 down to 0.8 what shows that the substitution of $(d - l_k)$ for l_{ar} is well justified.

3. Stability

The existence of steady-state solutions does not mean that the discharge with the corresponding parameters can occur over an extended period in time. For this case the discharge must be stable relative to fluctuations of the parameters, which are not controlled under real conditions. These parameters are the fluctuations of the cathode emission current, the charge multiplication factor in the CDR, the ion current emission from plasma to the CDR. As a result of these random processes the fluctuation of strong field region size occurs: $l_k \to l_k - \delta(l_k)$. It changes the field distribution in the CDR that affects the quantities γ and $w(x)$. The latter changes $K(l_k)$ and affects K_{ar}. The variation of these parameters at the size fluctuation l_k equals:

$$\delta(\gamma) = (\gamma - \theta)\, \delta(l_k)/l_k; \;\; \delta(e) = e\, \delta(l_k)/l_k;$$

$$\delta(w) \approx -w\, \delta(l_k)/l_k; \; \delta(K_{CDR}) \approx -K_{CDR}\, \delta(l_k)/l_k; \qquad (17)$$

$$\delta(K_{ar}) \approx -w\,\delta(l_k)\,[(l_{ar}/l_k) - 1],$$

where Eq. (15) was used for calculating γ. The total effect is described by variation of the product $\gamma K_\Sigma = \gamma[K_{CDR} + K_{ar}]$, which determines the fluctuation decay or the fluctuation effect amplification $\delta(l_k)$;

$$\delta(\gamma j_\Sigma) = \frac{\theta\, w(l_k)\, \delta(l_k)}{l_k}\left[\frac{\rho}{\theta} + (l_k - l_{ar}) - \frac{K_{CDR}(l_k)}{w(l_k)}\right]. \quad (18)$$

If $\delta(\gamma K_\Sigma) < 0$, then the criterion of independent development of the discharge is not performed and the variations, caused by the fluctuation, weaken. If $\delta(\gamma K_\Sigma) > 0$, then effect is reverse. In helium the coefficient θ is negative (see Eq. (6)) and $(\rho/\theta) = -1.55 \cdot 10^{-3}\, U_k/P$. This is a large quantity; at $U_k = 4$ kV and $P = 4$ kPa this quantity equals 1.55 mm that exceeds the electrode gap d in the EF generators.[1,2] From Fig. 1 it also follows that $K_{CDR}/w(l_k) \sim l_k$. Hence, it follows that under real conditions $\delta(\gamma K_\Sigma) > 0$, i.e., a discharge with the near-anode plasma is unstable and it will develop till the establishment of a quasistationary mode of anomalous glow discharge that is observed in practice.[13]

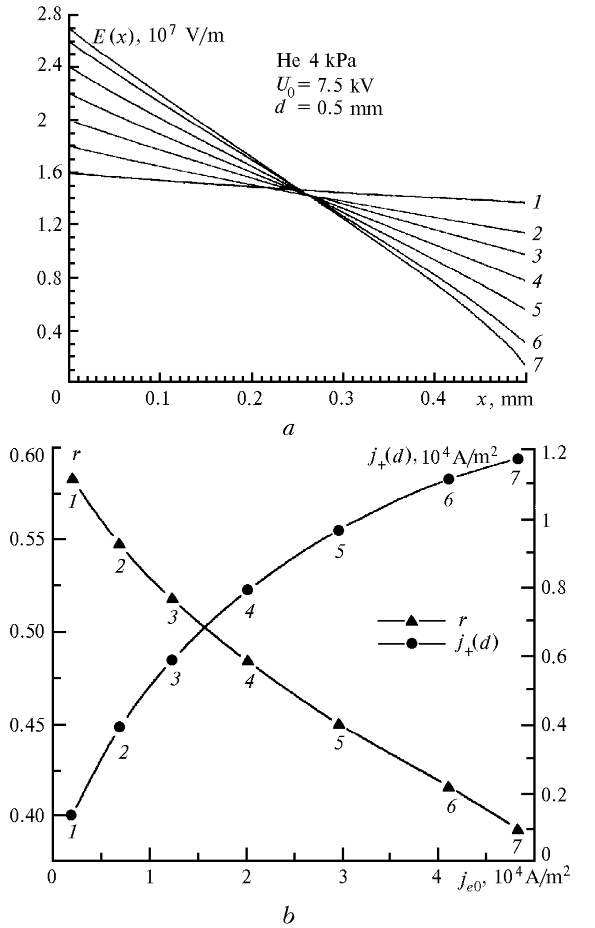

Fig. 2. Distribution of the electric field strength over the electrode gap length in a semi-self-maintained discharge (*a*), the value of the coefficient $j_+(d)/j_+(0)$, and the values required for the discharge maintenance the ion current density at the anode (*b*) calculated for some values of j_{e0}. Helium pressure equals 4 kPa, $U_0 = 7.5$ kV, $d = 0.5$ mm.

Quite different situation will be observed in the discharge in the absence of near-anode plasma when a strong field occupies the overall electrode gap (curves *1–4* in Fig. 1). This discharge exists only due to the ion current from the region beyond the anode occurring due to sagging of the field from the discharge gap or the emission from the plasma specially created here. In this case the relation (18) is not used to analyze the solution stability. The calculated discharge parameters at $l_k \geq d$ are shown in Fig. 2 and indicate that at control over ion current, injected through the anode, the discharge is stable and controllable. The experiments[14] confirmed that such a control is possible. Thus the existence of quasi-steady-state regime of high-voltage discharge in a gas at a medium pressure was found to be possible.

In conclusion we would state that the problem was solved in the quasi-stationary approximation. In the strong field region this state is achieved during the time required for an ion to cross it, that is, about 10 ns. The time needed to reach the equilibrium is maximum in the region of negative glow that is connected with the processes of charge storage for creating plasma with the equilibrium concentration and charge transfer in a weak electric field. Therefore there is enough time for the parameters of strong field region to fit the plasma state. Thus, Eqs. (1)–(6), describing the strong field region, are also applicable to the transient discharge phase. In this case the solutions give the CDR state at any moment in time.

4. Photoelectron discharge

Strictly speaking, the set of Eqs. (1)–(6) describes a glow discharge. However, it can easily be adapted for the case of a photoelectron discharge.[15] For this purpose the quantity j_{e0} is represented as a sum of two components: the current connected with the glow discharge mechanism $j_{e0}(\gamma_p)$ and the photoelectron current $j_{e0}(\gamma_v)$ generated by the UV illumination:

$$j_{e0} = j_{e0}(\gamma_p) + j_{e0}(\gamma_v), \quad (19)$$

where $j_{e0}(\gamma_p) = \gamma_p\, j_+(0)$; $j_{e0}(\gamma_v) = e\,\gamma_v\, q_{uv}$; e is the electron charge, γ_v is the coefficient of cathode photoelectron emission, q_{uv} is the flux of UV quanta from a source of auxiliary illumination incident on the cathode. It is evident that the photoelectron regime can take place only in the case when $j_{e0}(\gamma_v) \geq j_{e0}(\gamma_p)$. If assuming $\gamma_v \leq 0.1 \ll \gamma_p \sim 1$ (Refs. 9–11) we obtain $j_+(0) \ll e\, q_{uv}$. Hence, the photoelectron regime is possible only at a rather low ion current, otherwise, too high intensity of UV beam illumination is required. In other words, the best condition for it is the discharge stage shown in Fig. 2. In this case the strong field region occupies the entire cathode-anode gap, and the development of glow discharge slows down (is difficult) because $\gamma_p K(d) < 1$ and the discharge depends on ion current from the region beyond the anode.

Switching on of a UV illumination produces a photoelectron flow from the cathode. This supplementary electron current should be taken into account through the electron emission coefficient:

$$\gamma_\Sigma = \gamma_p + \gamma_v \, e \, q_{uv} / j_+(0). \quad (20)$$

On addition of the photoelectron component to γ_Σ we obtain the relationship $\gamma_\Sigma K(d) > 1$. Thus, the photoelectron discharge is a transient process and it is perceived as a phase of fast commutation at a breakdown of gaps at a high overvoltage.[11] Its specific features are the anomalously high value of the electron emission coefficient from the cathode and the resultant anomalously high electric field generation efficiency determined by the relation:

$$\eta = \gamma / (1 + \gamma). \quad (21)$$

Another feature is in a mechanism of CDR generation. In the glow discharges its parameters (voltage and dimensions) are determined by the ion flow balance to the emission boundary of plasma and from it to the CDR. In a photoelectron discharge this balance does not play that important role, because a cathode photoelectron flow is independent of the ion current. The CDR boundary is solely determined by the external field screening condition with a spatial ion charge generated by the photoelectrons. Therefore the dimension of the CDR in a photoelectron discharge in principle can be less than the CDR size in anomalous glow discharge.

During the photoelectron stage of the discharge development an electron beam creates plasma in the discharge gap thus increasing the ion current to the cathode. As a result the relationship between the components of electron current from the cathode $j_{e\,0}(\gamma_v) / j_{e\,0}(\gamma_\rho)$ starts to decrease. This finally leads to a decrease in the contribution from photoelectron mechanism to the discharge development and its development to the glow discharge regime.

Thus the above described method of calculation based on the function of ionization density $w(x)$ enabled us to determine the strong field region in an open discharge, i.e., to define the distribution of principal parameters over the discharge gap length, their interrelations and the dependence on the initial data. The calculated results have shown that regardless the low probability of a gas ionization by fast electrons, the gas ionization is a very essential factor and it cannot be neglected. The open discharge characteristics are shown to be caused both by the peculiarities of gas ionization by running away electrons and the geometry of a discharge chamber. The obtained pattern enabled us to propose an explanation of the photoelectron discharge role and mechanism of its occurrence and termination. A detailed description of the latter requires the extension of the developed method to the solution of unsteady-state problems.

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