# Formation of interference patterns in diffusely scattered fields by use of spatial filtering of the diffraction field of double-exposure quasi-Fourier and Fourier holograms. Part I

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The paper presents analysis of interferometer sensivity based on the double-exposure recording, by means of a negative lens, of a quasi-Fourier and Fourier holograms, to cross or longitudinal motions of a flat surface diffusely scattering incident light. It is shown that interference patterns are localized in the hologram plane and Fourier plane. For their recording, a spatial filtering of the diffraction field is needed. The experimental results obtained well agree with the theoretical ones.

Reference 1 shows that the double-exposure recording of the Fresnel hologram leads to formation of interference patterns, localized in the hologram plane and in the plane of the diffuser image formation, when cross and longitudinal motions of a flat surface diffusely scattering incident light are carried out before the photographic plate re-exposure. Recording these patterns while performing spatial filtering of the diffraction field in the corresponding planes provides an opportunity of the interferometer sensitivity determination both theoretically, and experimentally.

In its turn, as the investigation results have shown in Ref. 2, specificity of the double-exposure hologram recording of the diffuser in-focus image, connected, in particular, with recording of the subjective speckle-fields on a photographic plate, yields certain specific features in the formation of the interference patterns. Therefore, they are localized in the hologram plane and in a plane of the pupil image formation of a positive lens, by means of which the hologram recording was carried out. Moreover, the interferometer sensitivity to the diffuser cross motion depends both on the magnitude, and on the sign of the radius of curvature of a spherical wave front of coherent radiation used for illumination of flat light scattering surface at the stage of the hologram recording, at recording the interference pattern localized in the hologram plane. There is no such a dependence in the interference pattern localized in the plane of the pupil image formation of a positive lens. Besides, in the case of controlling the diffuser longitudinal motion, the interference pattern as the system of concentric fringes is formed only at its recording in the plane of the pupil image formation of a positive lens at a spatial filtering of the diffraction field being done in the hologram plane.

In this paper, I analyze specific features in the formation of interference patterns at the double-exposure recording by means of a negative lens of the quasi-Fourier and Fourier holograms in order to determine the interferometer sensitivity to cross or longitudinal motions of a flat surface diffusely scattering incident light. According to Fig. 1, a matte screen *1* that is in the plane $(x_1, y_1)$, is illuminated by a coherent radiation of a diverging spherical wave with the radius of curvature $R$.

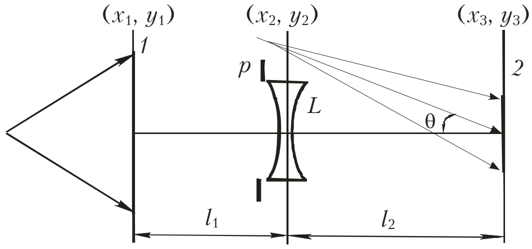


**Fig. 1.** Optical arrangement of the double-exposure recording of the quasi-Fourier hologram: *1* is the matte screen; *2* is the photographic plate; $L$ is the negative lens; $p$ is the aperture diaphragm.

The radiation diffusely scattered by the screen is recorded, after passing through a thin negative lens $L$ with a focal length $f$, by means of an off-axis reference wave on a photographic plate *2*. This photographic plate is in the plane $(x_3, y_3)$, during the first exposure, $\theta$ is the angle between the axis of the space-limited reference beam and the normal to a plane of the photographic plate. Before the repeated exposure, the matte screen is moved in its plane, for example, along the direction of the $x$-axis by the distance $a$.

In the Fresnel approximation and taking into account the beam spatial limitedness, distribution of the field complex amplitude, corresponding to the first exposure, in the subject channel in the plane of the photographic plate, is written as follows:

$$u_1(x_3,y_3) \sim \iiint\int_{-\infty}^{\infty} t(x_1,y_1)\exp\left[\frac{ik}{2R}\left(x_1^2+y_1^2\right)\right]\times$$

$$\times\exp\left\{\frac{ik}{2l_1}\left[(x_1-x_2)^2+(y_1-y_2)^2\right]\right\}\times$$

$$\times p(x_2,y_2)\exp\left[\frac{ik}{2f}\left(x_2^2+y_2^2\right)\right]\times$$

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$$\times\exp\left\{\frac{ik}{2l_2}\left[(x_2-x_3)^2+(y_2-y_3)^2\right]\right\}dx_1dy_1dx_2dy_2, \quad (1)$$

where $k$ is the wave number; $l_1$ is the distance between matte screen *1* and the principal plane $(x_2, y_2)$ of the lens $L$; $l_2$ is the distance between the planes $(x_2, y_2)$ and $(x_3, y_3)$; $t(x_1, y_1)$ is the complex amplitude of the diffuser transmission being a random function of coordinates; $p(x_2, y_2)$ is the pupil function[3] of the lens $L$.

Expression (1) can be presented in the following form:

$$u_1(x_3,y_3)\sim\exp\left[\frac{ik}{2l_2}(x_3^2+y_3^2)\right]\left\{\exp\left[-\frac{ikL_0}{2l_2^2}(x_3^2+y_3^2)\right]\times\right.$$

$$\left.\times\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\right\}\otimes P(x_3,y_3)\right\}, \quad (2)$$

where $\otimes$ is the convolution symbol; $L_0$ is the geometrical parameter of the optical system in the subject channel satisfying the condition $1/L_0 = 1/l_1 + 1/f + 1/l_2$; $1/l = 1/R + 1/l_1 - L_0/l_1^2$ is the introduced for the purpose of brevity; $F(x_3, y_3)$ is the Fourier image of the function $t(x_1, y_1)$ with the spatial frequencies $L_0x_3/\lambda l_1l_2$ and $L_0y_3/\lambda l_1l_2$; $\lambda$ is the wavelength of coherent light, used for the hologram recording and reconstruction; $P(x_3, y_3)$ is the Fourier image of the function $p(x_2, y_2)$ with the spatial frequencies $x_3/\lambda l_2$ and $y_3/\lambda l_2$.

If within the domain of existence of the function $P(x_3, y_3)$ (see Ref. 4), the phase change of a diverging spherical wave with the radius of curvature $l_2^2/L_0$ does not exceed $\pi$, this condition will hold for the region of the photographic plate with the diameter $D\le(dl_2/L_0)=d(1+l_2/l_1+l_2/f)$, where $d$ is the diameter of the $L$ pupil (see Fig. 1). Therefore, distribution of the complex field amplitude in the above plane $(x_3, y_3)$ is determined by the expression

$$u_1(x_3,y_3)\sim\exp\left[\frac{ik}{2r}(x_3^2+y_3^2)\right]\times$$

$$\times\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\otimes P(x_3,y_3)\right\}, \quad (3)$$

where $r=\dfrac{l_2^2}{l_2-L_0}=l_2+\dfrac{fl_1}{f+l_1}$ is the radius of curvature of a diverging spherical wave.

As follows from Eq. (3), the quasi-Fourier image of the function $t(x_1, y_1)$ is formed in the plane $(x_3, y_3)$, where every point within the limits of the circle with the diameter $D$ is smeared to the size of a subjective speckle determined by the width of $P(x_3, y_3)$ function, if the diameter $D_0$ of the illuminated area of the matte screen *1* (see Fig. 1) satisfies the condition $D_0\ge(dl_1/L_0)=d(1+l_1/l_2+l_1/f)$. Besides, the phase distribution of a diverging spherical wave with radius of curvature $r$ is superposed on the subjective speckle-field.

Distribution of the complex field amplitude, corresponding to the second exposure, in the subject channel in a plane of the photographic plate, can be written as follows

$$u_2(x_3,y_3)\sim\iiint\int_{-\infty}^{\infty}t(x_1+a,y_1)\exp\left[\frac{ik}{2R}(x_1^2+y_1^2)\right]\times$$

$$\times\exp\left\{\frac{ik}{2l_1}\left[(x_1-x_2)^2+(y_1-y_2)^2\right]\right\}\times$$

$$\times p(x_2,y_2)\exp\left[\frac{ik}{2f}(x_2^2+y_2^2)\right]\times$$

$$\times\exp\left\{\frac{ik}{2l_2}\left[(x_2-x_3)^2+(y_2-y_3)^2\right]\right\}dx_1dy_1dx_2dy_2, \quad (4)$$

which takes the following form:

$$u_2(x_3,y_3)\sim\exp\left[\frac{ik}{2r}(x_3^2+y_3^2)\right]\exp\left(\frac{ikL_0ax_3}{l_1l_2}\right)\times$$

$$\times\left\{F(x_3,y_3)\otimes\exp\left(-\frac{ikL_0ax_3}{l_1l_2}\right)\times\right.$$

$$\left.\times\left\{\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\otimes P(x_3,y_3)\right\}\right\}. \quad (5)$$

Since

$$\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\otimes\exp\left[\frac{iklL_0^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]=\delta(x_3,y_3),$$

where $\delta(x_3, y_3)$ is the Dirac delta function. As a result the integral representation of the convolution operation proves the identity

$$\exp\left[\frac{iklL_0^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\otimes\exp\left(-\frac{ikL_0ax_3}{l_1l_2}\right)\times$$

$$\times\left\{\exp\left[-\frac{ikL_0^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\otimes P(x_3,y_3)\right\}=$$

$$=\exp\left(-\frac{ika^2}{2l}\right)\exp\left(-\frac{ikL_0ax_3}{l_1l_2}\right)P\left(x_3+\frac{l_1l_2}{lL_0}a,y_3\right).$$

In view of this condition, distribution of the complex field amplitude, corresponding to the second exposure, in the subject channel in the plane of the photographic plate is determined by the expression

$$u_2(x_3,y_3)\sim\exp\left[\frac{ik}{2r}(x_3^2+y_3^2)\right]\exp\left(-\frac{ika^2}{2l}\right)\times$$

$$\times\exp\left(\frac{ikL_0ax_3}{l_1l_2}\right)\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\otimes\right.$$

$$\left.\otimes\exp\left(-\frac{ikL_0ax_3}{l_1l_2}\right)P\left(x_3+\frac{l_1l_2}{lL_0}a,y_3\right)\right\}. \quad (6)$$

It follows from expression (6) that, if compared with the distribution of the complex field amplitude (3), there occurs a shift of subjective speckles by $al_1l_2/lL_0$, and their tilt by the angle $aL_0/l_1l_2$.

Provided that the double-exposure recording of the quasi-Fourier hologram is performed with a diverging spherical wave with the radius of wave front curvature $r$ within the linear portion of the blackening curve of the photographic material, distribution of the complex amplitude of the hologram transmission, corresponding to the (–1)st diffraction order takes, based on Eqs. (3) and (6), the following form

$$\tau(x_3,y_3) \sim \exp(-ikx_3\sin\theta)\times$$
$$\times\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\otimes\right.$$
$$\otimes P(x_3,y_3)+\exp\left(-\frac{ika^2}{2l}\right)\exp\left(\frac{ikL_0ax_3}{l_1l_2}\right)\times$$
$$\times\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\otimes\right.$$
$$\left.\left.\otimes\exp\left(-\frac{ikL_0ax_3}{l_1l_2}\right)P\left(x_3+\frac{l_1l_2}{lL_0}a,y_3\right)\right\}\right\}. \quad (7)$$

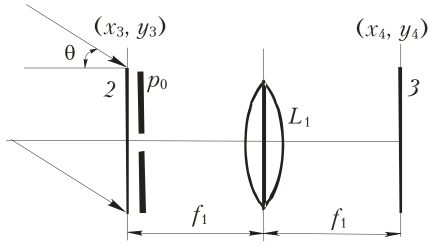



**Fig. 2.** Optical arrangement of the interference pattern recording, localized in a plane of the diffuser image formation: *2* is the hologram; *3* is the plane of the hologram recording; $L_1$ is the positive lens; $p_0$ is the spatial filter.

Let the spatial filtering of the diffraction field be carried out at stage of the hologram reconstruction (Fig. 2) by means of a round aperture in an opaque screen $p_0$ in its plane on the optical axis. Thus, within the limits of a filtering aperture diameter, the phase change $(kL_0ax_3/l_1l_2)$ does not exceed $\pi$. Then distribution of the complex field amplitude at the exit of a spatial filter is determined by the expression

$$u(x_3,y_3) \sim p_0(x_3,y_3)\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\otimes\right.$$
$$\otimes\left[P(x_3,y_3)+\exp\left(-\frac{ika^2}{2l}\right)\exp\left(-\frac{ikL_0ax_3}{l_1l_2}\right)\times\right.$$
$$\left.\left.\times P\left(x_3+\frac{l_1l_2}{lL_0}a,y_3\right)\right]\right\}, \quad (8)$$

where $p_0(x_3, y_3)$ is the transmission function of the opaque screen with a round aperture.[5]

For the sake of brevity, let us assume, here and below, that for a positive lens $L$, $f_1$ equals $l_2$. Then because of the function $p(x_2, y_2)$ parity, distribution of the complex field amplitude in the focal plane $(x_4, y_4)$ of $L_1$ takes the following form

$$u(x_4,y_4) \sim \left\{p(x_4,y_4)t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\exp\left[\frac{ikl_1^2}{2lL_0^2}(x_4^2+y_4^2)\right]+\right.$$
$$+p\left(x_4+\frac{L_0}{l_1}a,y_4\right)\exp\left(\frac{ikl_1ax_4}{lL_0}\right)\times$$
$$\times\exp\left(\frac{ika^2}{2l}\right)t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\times$$
$$\left.\times\exp\left[\frac{ikl_1^2}{2lL_0^2}(x_4^2+y_4^2)\right]\right\}\otimes P_0(x_4,y_4), \quad (9)$$

where $P_0(x_4, y_4)$ is the Fourier image of the function $p_0(x_3, y_3)$ with the spatial frequencies $x_4/\lambda l_2$ and $y_4/\lambda l_2$.

If within the limits of overlap of two images $p(x_4, y_4)$ and $p(x_4 + aL_0/l_1, y_4)$ of the lens $L$ pupil (see Fig. 1), the period of function variation $1+\exp(ika^2/2l)\exp(ikl_1ax_4/lL_0)$ exceeds at least by an order of magnitude (see Ref. 6) the function $P_0(x_4, y_4)$ width, we shall remove it from the convolution integration sign in Eq. (9). So, the illumination distribution in the focal plane $(x_4, y_4)$ of $L_1$ (see Fig. 2) is determined, because of smallness of $L_0a/l_1$ value, by the expression

$$I(x_4,y_4) \sim \left[1+\cos\left(\frac{kl_1}{lL_0}ax_4+\frac{ka^2}{2l}\right)\right]\times$$
$$\times\left|p(x_4,y_4)t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\times\right.$$
$$\left.\times\exp\left[\frac{ikl_1^2}{2lL_0^2}(x_4^2+y_4^2)\right]\otimes P_0(x_4,y_4)\right|^2. \quad (10)$$

As follows from Eq. (10), the subjective speckle structure was modulated by the interference patterns periodically varying along the $x$-axis, in the plane of the diffuser image formation, limited by the size of the lens $L$ pupil (see Fig. 1). Measurement of periods of interference fringes for the known values of $\lambda$, $l_1$, $l$, and $L_0$ provides the possibility of measuring the cross motion of the flat surface diffusely scattering incident light.

Let the spatial filtering of the diffraction field be carried out on the optical axis in the plane $(x_4, y_4)$ (Fig. 3) of the diffuser image formation, at the stage of the quasi-Fourier hologram reconstruction.

In this case, based on the integral representation of the convolution operation, distribution of the complex field amplitude in the hologram exit, can be written as follows

$$u(x_3,y_3) \sim \exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\left\{\exp\left[\frac{iklL_0^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\otimes\right.$$

$$\otimes t\left(\frac{lL_0}{l_1l_2}x_3,\frac{lL_0}{l_1l_2}y_3\right)\otimes P(x_3,y_3)+$$

$$+\exp\left(-\frac{ika^2}{2l}\right)\exp\left(\frac{ikL_0ax_3}{l_1l_2}\right)\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\times$$

$$\times\left\{\exp\left[\frac{iklL_0^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes t\left(\frac{lL_0}{l_1l_2}x_3,\frac{lL_0}{l_1l_2}y_3\right)\right\}\otimes$$

$$\otimes\exp\left(-\frac{ikL_0ax_3}{l_1l_2}\right)P\left(x_3+\frac{l_1l_2}{lL_0}a,y_3\right). \quad (11)$$

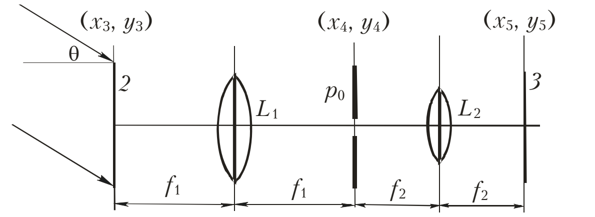



**Fig. 3.** Optical arrangement of the interference pattern recording, localized in the hologram plane: *2* is the hologram; *3* is the plane of the interferogram recording; $L_1$ and $L_2$ are the positive lenses; $p_0$ is the spatial filter.

Ignoring the field spatial limitedness, distribution of its complex amplitude in the plane $(x_4, y_4)$ is determined by the expression

$$u(x_4,y_4)\sim p(x_4,y_4)\left\{\exp\left[\frac{ikl_1^2}{2lL_0^2}\left(x_4^2+y_4^2\right)\right]\otimes\right.$$

$$\left.\otimes F_1(x_4,y_4)\exp\left[-\frac{ikl_1^2}{2lL_0}\left(x_4^2+y_4^2\right)\right]\right\}+$$

$$+p\left(x_4+\frac{L_0}{l_1}a,y_4\right)\exp\left(\frac{ikl_1ax_4}{lL_0}\right)\exp\left(\frac{ika^2}{2l}\right)\times$$

$$\times\left\{\exp\left\{\frac{ikl_1^2}{2lL_0^2}\left[\left(x_4-\frac{L_0}{l_1}a\right)^2+y_4^2\right]\right\}\otimes\right.$$

$$\left.\otimes F_1(x_4,y_4)\exp\left[-\frac{ikl_1^2}{2lL_0^2}\left(x_4^2+y_4^2\right)\right]\right\}, \quad (12)$$

where $F_1(x_4, y_4)$ is the Fourier image of the function $t\left(\frac{lL_0}{l_1l_2}x_3,\frac{lL_0}{l_1l_2}y_3\right)$ with the spatial frequencies $x_4/\lambda l_2$ and $y_4/\lambda l_2$.

If, within the limits of the filtering aperture diameter of the spatial filter $p_0$ (see Fig. 3), the phase change $(kl_1ax_4/lL_0)\leq\pi$, then the distribution of the complex field amplitude in its exit takes the following form

$$u(x_4,y_4)\sim p_0(x_4,y_4)\left\{\exp\left[\frac{ikl_1^2}{2lL_0^2}\left(x_4^2+y_4^2\right)\right]\otimes\right.$$

$$\left.\otimes F_1(x_4,y_4)\exp\left[-\frac{ikl_1^2}{2lL_0^2}\left(x_4^2+y_4^2\right)\right]\right\}+$$

$$+\exp\left(\frac{ika^2}{2l}\right)\left\{\exp\left\{\frac{ikl_1^2}{2lL_0^2}\left[\left(x_4-\frac{L_0}{l_1}a\right)^2+y_4^2\right]\right\}\otimes\right.$$

$$\left.\left.\otimes F_1(x_4,y_4)\exp\left[-\frac{ikl_1^2}{2lL_0^2}\left(x_4^2+y_4^2\right)\right]\right\}\right\}. \quad (13)$$

Let us assume, for brevity, that the focal lengths are equal $f_2 = f_1 = l_2$ for $L_2$ (see Fig. 3). Then in the focal plane $(x_5, y_5)$, the distribution of the complex field amplitude is determined by the expression

$$u(x_5,y_5)\sim\left\{\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}\left(x_5^2+y_5^2\right)\right]\right]\left\{\exp\left[\frac{iklL_0^2}{2l_1^2l_2^2}\left(x_5^2+y_5^2\right)\right]\otimes\right.$$

$$\left.\otimes t\left(-\frac{lL_0}{l_1l_2}x_5,-\frac{lL_0}{l_1l_2}y_5\right)\right\}+\exp\left(\frac{ika^2}{2l}\right)\exp\left(-\frac{ikL_0ax_5}{l_1l_2}\right)\times$$

$$\times\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}\left(x_5^2+y_5^2\right)\right]\left\{\exp\left[\frac{iklL_0^2}{2l_1^2l_2^2}\left(x_5^2+y_5^2\right)\right]\otimes\right.$$

$$\left.\left.\otimes t\left(-\frac{lL_0}{l_1l_2}x_5,-\frac{lL_0}{l_1l_2}y_5\right)\right\}\right\}\otimes P_0(x_5,y_5), \quad (14)$$

where $P_0(x_5, y_5)$ is the Fourier image of the transmission function $p_0(x_4, y_4)$ of the spatial filter with the spatial frequencies $x_5/\lambda l_2$ and $y_5/\lambda l_2$.

If the period of function variation $1+\exp(ika^2/2l)\times\exp(-ikL_0ax_3/l_1l_2)$ exceeds, at least by an order of magnitude, the function $P_0(x_5, y_5)$ width, which determines the size of subjective speckle in the recording plane *3* (see Fig. 3), we shall remove it in Eq. (14) from the convolution integral sign. Moreover, we shall use the integral representation of the convolution operation. Therefore, the illumination distribution in the plane $(x_5, y_5)$ takes the following form

$$I(x_5,y_5)\sim\left[1+\cos\left(\frac{kL_0ax_5}{l_1l_2}-\frac{ka^2}{2l}\right)\right]\times$$

$$\times\left|F(-x_5,-y_5)\otimes\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}\left(x_5^2+y_5^2\right)\right]\otimes P_0(x_5,y_5)\right|^2. \quad (15)$$

According to Eq. (15), in constructing the hologram image by means of an optical system like Keplerian telescope accompanied by a spatial filtering of the diffraction field, the interference pattern is formed in the form of interference fringes periodically varying along the $x$-axis and modulating the subjective speckle structure in the recording plane *3* (see Fig. 3). Thus, the frequency of interference fringes does not depend on the radius of curvature of a spherical wave front of a coherent radiation used for the diffuser illumination at the stage of the hologram recording. Besides, when diameter of the collimated beam of a coherent radiation, reconstructing the hologram, exceeds the value of $D$ and the $L_1$ diameter (see Fig. 3) also exceeds this value, the spatial extension of the interference pattern is limited by the area of the quasi-Fourier image of the function $t(x_1, y_1)$ in the hologram plane.

As follows from comparison of the expressions (10) and (15), for the interference pattern localized in the plane of the diffuser image formation, the interferometer sensitivity to the cross motion of the diffuser surface changes by $G_1 = l_2(l_1 - L_0)/L_0^2 + l_2 l_1^2/(L_0^2 R)$ times. Thus, at the fixed values of $l_1$, $l_2$, and $L_0$, it increases with reduction of $R$ due to the increase of uniform motion of subjective speckles, corresponding to the second exposure, in the hologram plane.

At illumination of matte screen *1* (see Fig. 1) by a coherent radiation of a converging wave with the radius of curvature $R$ in the above analysis of formation of the interference patterns characterizing the diffuser cross motion, it is necessary to replace the value of $l$ by the value, satisfying the condition $1/l = -1/R + 1/l_1 - L_0/l_1^2$. In this case, the interferometer sensitivity changes by $G_2 = l_2(l_1 - L_0)/L_0^2 - l_2 l_1^2/(L_0^2 R)$ times. Thus, for the fixed values of $l_1$, $l_2$, and $L_0$, it decreases with reduction of $R$ in the interval $l_1^2/(l_1 - L_0) \le R \le \infty$ down to zero, when $R$ equals $l_1^2/(l_1 - L_0)$ (condition of the Fourier hologram recording[7]), and in the hologram plane, the "frozen" interference fringes are localized due to the speckle motion. Further reduction of the value $R$ leads to the interferometer sensitivity increase owing to emergence and increase of a uniform motion in the hologram plane of the subjective speckles, corresponding to the second exposure. As an example, the dependences $G_1$ and $G_2$ on $R$ for the fixed values of $f$ = 220 mm, $l_1$ = 200 mm, $l_2$ = 160 mm are presented in Fig. 4.

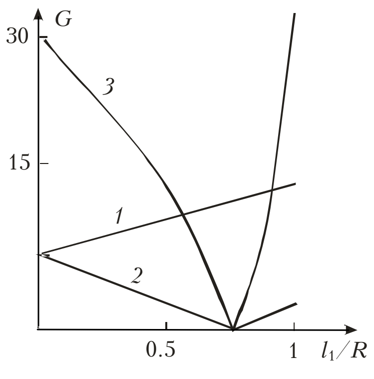



**Fig. 4.** Dependences of $G$ coefficients of the interferometer sensitivity for the fixed values of $f$ = 220 mm, $l_1$ = 200 mm, $l_2$ = 160 mm: $G_1$ (*1*); $G_2$ (*2*); $G_3$ (*3*).

From analysis of formation of the interference patterns, characterizing the cross motion of flat diffuser surface, when the double-exposure recording of the quasi-Fourier hologram is being carried out with the use of a negative lens, it follows that they are localized in two planes: in the hologram plane and in the far-field zone of diffraction, where the diffuser image is formed. It accounts for the fact that, on the one hand, in the hologram plane, there is a motion of subjective speckles by the identical distance, corresponding to the second exposure, relative to the speckles of the first exposure. Thus, their overlap at the stage of the hologram reconstruction is achieved in the far-field zone of diffraction. On the other hand, the tilt angle that exists in the hologram plane of subjective speckles, corresponding to the second exposure, relative to the identical speckles of the first exposure, conditions the localization of the interference pattern in it. Moreover, spatial filtering of the diffraction field in the corresponding planes provides a possibility of determining the interferometer sensitivity to the cross motion for the interference patterns recorded. Let us now, before the repeated exposure of the photographic plate *2* (see Fig. 1), the matte screen *1* be moved along the $z$-axis by the distance of $\Delta l \ll l_1 R$. As a result, in the used approximation, the complex amplitude distribution of the hologram transmission, corresponding to the (–1)st order of diffraction, takes the following form

$$\tau'(x_3,y_3) \sim \exp(-ikx_3\sin\theta)\times \left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes P(x_3,y_3) + \exp(ik\Delta l)\exp\left[-\frac{ik\Delta lL_0^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\times \left\{F'(x_3,y_3)\otimes\exp\left[-\frac{ikl'L_0'^2}{2(l_1+\Delta l)^2 l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes P(x_3,y_3)\right\}\right\}, \quad (16)$$

where

$$L_0' = L_0\left(1+\frac{L_0\Delta l}{l_1^2}\right),\ \frac{1}{l'} = \frac{1}{R-\Delta l} + \frac{L_0'}{(l_1+\Delta l)^2} + \frac{1}{l_1+\Delta l}$$

are the symbols introduced for brevity, $F'(x_3, y_3)$ is the Fourier image of the function $t(x_1, y_1)$ with the spatial frequencies $\frac{L_0' x_3}{\lambda(l_1+\Delta l)l_2}$, and $\frac{L_0' y_3}{\lambda(l_1+\Delta l)l_2}$.

Expression (16) demonstrates that subjective speckles, corresponding to the second exposure, are moved in the hologram plane along the radius from the optical axis relative to the speckles of the first exposure due to the difference in the scales of the Fourier images of the function $t(x_1, y_1)$. Moreover, the variation along the radius from the optical axis of their tilt angle conditions decorrelation of the subjective speckle structures of the two exposures. Besides, the factor $\exp\left[-ikL^2\Delta l(x_3^2+y_3^2)/(2l_1^2l_2^2)\right]$ characterizes the varying tilt angle of subjective speckles, corresponding to the second exposure, relative to the speckles of the first exposure.

If at the stage of the double-exposure quasi-Fourier hologram reconstruction in its plane on the optical axis, a spatial filtering of the diffraction field is carried out, then assume that within the limits of the filtering aperture diameter of the spatial filter $p_0$ (see Fig. 2), the phase change $kL_0^2\Delta l(x_3^2+y_3^2)/(2l_1^2 l_2^2)$ does not exceed $\pi$. In addition, consider that diameter $d_f$ of a filtering aperture satisfies the condition $d_f \le 2\lambda l_1^2 l_2/d(l_1-L_0)\Delta l$. Then, at the exit of a spatial filter, distribution of the field complex amplitude is determined by the following expression

$$u'(x_3,y_3) \sim p_0(x_3,y_3)\left\{F(x_3,y_3)\otimes \exp\left[-\frac{iklL_0^2}{2l_1^2 l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes\right.$$
$$\otimes P_0(x_3,y_3)+\exp(ik\Delta l)\times$$
$$\left.\times\left\{F(x_3,y_3)\otimes\exp\left[-\frac{ikl'L_0^2}{2l_1^2 l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes P(x_3,y_3)\right\}\right\}. \quad (17)$$

After the Fourier transform, one obtains distribution of the complex field amplitude in the plane $(x_4, y_4)$ (see Fig. 2) in the following form

$$u'(x_4,y_4) \sim \left\{p(x_4,y_4)\,t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\times\right.$$
$$\times\exp\left[\frac{ikl_1^2}{2lL_0^2}\left(x_4^2+y_4^2\right)\right]+p(x_4,y_4)\exp(ik\Delta l)\times$$
$$\times t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\exp\left[\frac{ikl_1^2}{2lL_0^2}\left(x_4^2+y_4^2\right)\right]\times$$
$$\left.\times\exp\left[\frac{ikM\Delta l}{2l_1^2}\left(x_4^2+y_4^2\right)\right]\right\}\otimes P_0(x_4,y_4), \quad (18)$$

where $M=\dfrac{l_1^4-R^2(l_1-L_0)^2}{R^2L_0^2}$ is the symbol introduced for brevity.

If the period of the function variation $1+\exp(ik\Delta l)\times$ $\times\exp\left[ikM\Delta l(x_4^2+y_4^2)/2l_1^2\right]$ exceeds at least by an order of magnitude the function $P_0(x_4, y_4)$ width, we shall remove it in Eq. (18) from the convolution sign. Then distribution of illumination in the recording plane *3* (see Fig. 2) is determined by the following expression

$$I'(x_4,y_4)\sim\left\{1+\cos\left[k\Delta l+\frac{kM\Delta l}{2l_1^2}\left(x_4^2+y_4^2\right)\right]\right\}\times$$
$$\times\left|p(x_4,y_4)\,t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\times\right.$$
$$\left.\times\exp\left[\frac{ikl_1^2}{2lL_0^2}\left(x_4^2+y_4^2\right)\right]\otimes P_0(x_4,y_4)\right|^2. \quad (19)$$

Expression (19) shows that in the plane of the diffuser image formation, limited by the size of the pupil $L$ (see Fig. 1), the subjective speckle structure is modulated by the bands of equal tilt, that is, by the system of concentric interference fringes. Moreover, measurement of their radii in the adjacent orders of interference provides a possibility of determining longitudinal motion of the flat diffuser surface having known values of $\lambda$, $R$, $L_0$, and $l_1$.

Let at the stage of the double-exposure quasi-Fourier hologram reconstruction, characterizing the diffuser longitudinal motion, a spatial filtering of the diffraction field be carried out on the optical axis in the plane $(x_4, y_4)$ (see Fig. 3) of the diffuser image formation. In this case, ignoring the field spatial limitedness, distribution of its complex amplitude takes the following form

$$u'(x_4,y_4)\sim p(x_4,y_4)\,t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\exp\left[\frac{ikl_1^2}{2lL_0^2}\left(x_4^2+y_4^2\right)\right]+$$
$$+\exp(ik\Delta l)\exp\left[\frac{ikl_1^2}{2\Delta lL_0^2}\left(x_4^2+y_4^2\right)\right]\otimes p(x_4,y_4)\times$$
$$\times t\left(-\frac{l_1+\Delta l}{L_0'}x_4,-\frac{l_1+\Delta l}{L_0'}y_4\right)\exp\left[\frac{ik(l_1+\Delta l)^2}{2l'L_0'^2}\left(x_4^2+y_4^2\right)\right]. \quad (20)$$

The filtering aperture diameter of a spatial filter $p_0$ (see Fig. 3) does not exceed $2\lambda l_1^2 l_2/d(l_1-L)\Delta l$ and the phase change satisfies the condition $\left[kM\Delta l(x_4^2+y_4^2)/2l_1^2\right]\le\pi$. Therefore, because the function $\exp\left[\dfrac{ikl_1^2}{2\Delta lL_0^2}\left(x_4^2+y_4^2\right)\right]$ is close, by the order of magnitude, to the Dirac delta function, distribution of the complex field amplitude at the exit of a spatial filter is determined by the following expression

$$u'(x_4,y_4)\sim p_0(x_4,y_4)\left\{t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\times\right.$$
$$\times\exp\left[\frac{ikl_1^2}{2lL_0^2}\left(x_4^2+y_4^2\right)\right]+\exp(ik\Delta l)\exp\left[\frac{ikl_1^2}{2\Delta lL_0^2}\left(x_4^2+y_4^2\right)\right]\otimes$$
$$\left.\otimes t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\exp\left[\frac{ikl_1^2}{2lL_0^2}\left(x_4^2+y_4^2\right)\right]\right\}. \quad (21)$$

Having made the Fourier transform one obtains distribution of the complex field amplitude in the plane $(x_5, y_5)$ (see Fig 3) in the following form

$$u'(x_5,y_5)\sim\left\{\left\{1+\exp(ik\Delta l)\exp\left[-\frac{ikL_0^2\Delta l}{2l_1^2l_2^2}\left(x_5^2+y_5^2\right)\right]\right\}\times\right.$$
$$\left.\times\left\{F(-x_5,-y_5)\otimes\exp\left[-\frac{iklL_0^2}{2l_1^2l_2^2}\left(x_5^2+y_5^2\right)\right]\right\}\right\}\otimes P_0(x_5,y_5). \quad (22)$$

If the period of function $1+\exp(ik\Delta l)\times$ $\times\exp\left[-\dfrac{ikL_0^2\Delta l}{2l_1^2l_2^2}\left(x_5^2+y_5^2\right)\right]$ exceeds, at least by an order of magnitude, the function $P_0(x_5, y_5)$ width, which determines the size of subjective speckle in the recording plane *3* (see Fig. 3), we shall remove it in Eq. (22) from the convolution integral. Then the distribution of illumination over the recording plane $(x_5, y_5)$ is determined by the following expression

$$I'(x_5,y_5) \sim \left\{1+\cos\left[k\Delta l - \frac{kL_0^2\Delta l}{2l_1^2 l_2^2}\left(x_5^2+y_5^2\right)\right]\right\}\times$$

$$\times\left|F(-x_5,-y_5)\otimes\exp\left[-\frac{iklL_0^2}{2l_1^2 l_2^2}\left(x_5^2+y_5^2\right)\right]\otimes P_0(x_5,y_5)\right|^2. \quad (23)$$

According to Eq. (23), in a plane of the hologram image formation, the subjective speckle structure is modulated by the bands of equal tilt that is by the system of concentric fringes. Thus, their radii do not depend on the radius of curvature of the spherical wave front of the coherent radiation used for illumination of matte screen *1* (see Fig. 1) at stage of the hologram recording. Besides, because of the above stated conditions, the spatial extension of the interference pattern is limited by the domain of existence of quasi-Fourier image of the function $t(x_1, y_1)$, like in case of monitoring the diffuser cross motion.

As follows from comparison of the expressions (19) and (23), for the interference pattern localized in the plane of the diffuser image formation, the interferometer sensitivity to cross motion of a flat surface diffusely scattering incident light changes by $G_3 = Ml_2^2/L_0^2$ times. Thus, it is equal to zero, when $R$ equals $l_1^2/(l_1 - L_0)$, and its dependence on $R$ at the fixed values of $f$, $l_1$, and $l_2$ is presented in Fig. 4.

If the matte screen *1* is illuminated (see Fig. 1) by a coherent radiation of a converging spherical wave with the radius of the wave front curvature $R$, then to analyze the interference pattern formation characterizing the diffuser cross motion, one has to change $l$ by the quantity satisfying the condition $1/l = -1/R + 1/l_1 - L_0/l_1^2$, and $l'$ by the quantity satisfying the condition

$$\frac{1}{l'} = -\frac{1}{R+\Delta l} + \frac{1}{l_1+\Delta l} - \frac{L_0'}{(l_1+\Delta l)^2}.$$

In this case, the interferometer sensitivity changes by $G_3$ times for the interference pattern, localized in the plane of the diffuser image formation. Moreover, in comparison with the diffuser cross motion monitoring at the double-exposure Fourier hologram recording when radius of curvature of a converging spherical wave front of a coherent radiation used for the diffuser illumination is equal to $l_1^2/(l_1 - L_0)$, the recording of the interference pattern localized in the hologram plane needs for a spatial filtering of the diffraction field to be done. This is explained by the fact that subjective speckles in the hologram plane corresponding to the second exposure are shifted along the radius from the optical axis relative to the speckles of the first exposure.

As follows from analysis of formation of the interference patterns characterizing the axial motion of the diffuser surface, when a negative lens is used to record two-exposure Fourier holograms, the patterns are localized in two planes: in the hologram plane and in the far-field zone of the diffraction where the diffuser image is formed. This is explained by the facts that, on the one hand, there is a tilt angle varying along the radius from the optical axis, in the hologram plane, of the subjective speckles of the second exposure relative to the speckles of the first exposure. As a result, in a plane of the hologram image formation, the interference pattern is formed, provided that a spatial filtering of the diffraction field is being performed, which enables one to obtaining identical speckles of two exposures in the plane of the interferogram recording. On the other hand, owing to specific orientation of the subjective speckles in the hologram plane is such that there is an additional variation of the tilt angle of the subjective speckles of the second exposure with respect to the speckles of the first exposure along the radius from the optical axis. In this case, obtaining of identical speckles of two exposures is reached in the Fourier plane if spatial filtering of the diffraction field is performed in the hologram plane.

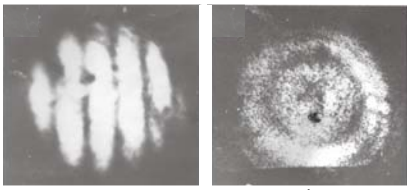
In the experiment, the double-exposure quasi-Fourier and Fourier holograms were recorded on the photographic plates of Mikrat-VRL type using radiation of a He–Ne laser at the wavelength of 0.63 μm. The technique of experimental investigations consisted in comparing the holograms recorded at fixed values of the cross $a = (0.05 \pm 0.002)$ mm and axial $\Delta l = (1 \pm 0.002)$ mm motions. The distances $l_1$ and $l_2$ were equal to 200 and 160 mm, respectively. The focal length of a negative lens was $f = 220$ mm, and the pupil diameter $d = 14$ mm. The diameter of illuminated area on the matte screen was equal to 60 mm. For the reference beam, angle θ equaled 10° and $r$ was equal to 265 mm. Various radii of curvature of diverging or converging spherical wave fronts of coherent radiation for illumination of the matte screen were chosen in the limits from $R = \infty$ down to $|R| = 200$ mm.

Figure 5 presents the interference patterns, localized in a plane of the diffuser image formation and characterizing the screen cross motion. A mark in the form of the letter "T" was preliminary drawn on the matte screen, and the Russian letter "Л" – was drawn on the lateral surface of a negative lens.

*a* *b* *c*

**Fig. 5.** Interference patterns, localized in the plane of the diffuser image formation characterizing its cross motion: the diffuser illumination by a collimated beam (*a*); by a diverging wave (*b*); by a converging wave (*c*).

The interference patterns were recorded while performing spatial filtering of the diffraction field in the hologram plane at its reconstruction by use of a small-aperture (≈ 2 mm) laser beam. In Fig. 5*a,* the screen *1* (see Fig. 1) is illuminated by a collimated beam, in Fig. 5*b,* the matte screen is illuminated by radiation of a diverging spherical wave with $R$ equal 200 mm, in Fig. 5*c,* it is illuminated with a converging spherical wave with $R$ equal 200 mm. In these three cases, as well as in the subsequent ones, connected with the change of magnitude and sign of $R$, the interference patterns, localized in the hologram plane, had identical frequency of interference fringes, corresponding to the frequency presented in Fig. 6*a*.



*a* *b*

**Fig. 6.** Interference patterns, localized in the hologram plane and characterizing: cross (*a*) and longitudinal motion of the diffuser (*b*).

The recording of interference patterns shown in Fig. 6 was carried out at hologram illumination (see Fig. 3) by a collimated beam of 60-mm diameter while performing spatial filtering of the diffraction field in the focal plane of the lens $L_1$ (see Fig. 3) having the diameter of 65 mm. The spatial extension of the interference pattern, localized in the hologram plane was 34 mm and corresponded, like that in Ref. 7, to the calculated value.

By measuring periods of interference fringes, the coefficients of $G_1$ and $G_2$ were determined (apart from the fact that they can be determined from the measurements of $f$, $l_1$, $l_2$, and $R$). The values of $G_1$ and $G_2$ obtained correspond to Fig. 4 within the experimental error of 10%.

Let, in case of the diffuser cross motion monitoring, the spatial filtering of the diffraction field be carried out in the hologram plane off of the optical axis, i.e., we assume that the center of the filtering aperture has the coordinates $x_{03}$, 0. Because its diameter is much larger that the size of the function $P(x_3, y_3)$ domain of existence, and taking into account the condition that within the limits of the filtering aperture of the spatial filter $p_0$ (see Fig. 2), the phase change is $(kL_0ax_3/l_1l_2) \le \pi$, the distribution of the complex field amplitude in the exit of the spatial filter takes the following form

$$u(x_3,y_3) \sim p_0(x_3,y_3)\Bigg\{F(x_3+x_{03},y_3)\otimes$$

$$\otimes\exp\left\{-\frac{iklL_0^2}{2l_1^2l_2^2}\left[(x_3+x_{03})^2+y_3^2\right]\right\}\otimes$$

$$\otimes\exp\left(\frac{ikL_0x_{03}x_3}{l_1l_2}\right)P(x_3,y_3)+\exp\left(\frac{ikL_0ax_{03}}{l_1l_2}\right)\times$$

$$\times F(x_3+x_{03},y_3)\otimes\exp\left[-\frac{ikL_0a}{l_1l_2}(x_3+x_{03})\right]\times$$

$$\times\exp\left\{-\frac{iklL_0^2}{2l_1^2l_2^2}\left[(x_3+x_{03})^2+y_3^2\right]\right\}\otimes$$

$$\otimes\exp\left[-\frac{ikL_0a}{l_1l_2}(x_3+x_{03})\right]\exp\left(\frac{ikL_0x_{03}x_3}{l_1l_2}\right)P(x_3,y_3)\Bigg\}. \quad (24)$$

In the considered case, after making the Fourier transform, the distribution of the complex field amplitude in the plane $(x_4, y_4)$ (see Fig. 2) is determined by the expression

$$u(x_4,y_4)\sim\left\{p\left(x_4-\frac{L_0}{l_1}x_{03},y_4\right)t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\times\right.$$

$$\times\exp\left[\frac{ikl_1^2}{2lL_0^2}\left(x_4^2+y_4^2\right)\right]\exp\left(\frac{i2kx_{03}x_4}{l_2}\right)+$$

$$+p\left(x_4-\frac{L_0}{l_1}x_{03}+\frac{L_0}{l_1}a,y_4\right)t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\times$$

$$\times\exp\left\{\frac{ikl_1^2}{2lL_0^2}\left[\left(x_4+\frac{L_0}{l_1}a\right)^2+y_4^2\right]\right\}\exp\left(\frac{i2kx_{03}x_4}{l_2}\right)\times$$

$$\left.\times\exp\left(\frac{ikL_0ax_{03}}{l_1l_2}\right)\right\}\otimes P_0(x_4,y_4), \quad (25)$$

based on which light distribution in the recording plane *3* (see Fig. 2) takes the following form

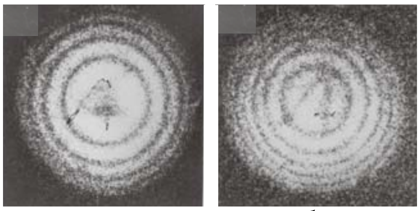
$$I(x_4,y_4)\sim\left\{1+\cos\left[\frac{kl_1a}{lL_0}\left(x_4+\frac{lL_0^2}{l_1^2l_2}x_{03}\right)+\frac{k}{2l}a^2\right]\right\}\times$$

$$\times\left|p\left(x_4-\frac{L_0}{l_1}x_{03},y_4\right)t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\times\right.$$

$$\left.\times\exp\left[\frac{ikl_1^2}{2lL_0^2}\left(x_4^2+y_4^2\right)\right]\exp\left(\frac{i2kx_{03}x_4}{l_2}\right)\otimes P_0(x_4,y_4)\right|^2. \quad (26)$$

As follows from the comparison of Eqs. (10) and (26), at motion of the filtering aperture along the $x$-axis, the motion of interference pattern occurs with respect to the static image of the flat surface diffusely scattering incident light. Besides, in the process of $x_{03}$ variation, the interference pattern phase changes from 0 to π, when the center of the filtering aperture moves from the minimum of the interference pattern, localized in the hologram plane, to its maximum ("living" interference fringes).

The interference patterns shown in Fig. 7 are localized in the plane of the diffuser image formation and characterize its longitudinal motion, when at the stage of the hologram recording, the matte screen *1* (see Fig. 1) is illuminated by a collimated beam (Fig. 7*a*); by radiation of a converging spherical wave with the radius of the wave front curvature of 200 mm (Fig. 7*b*). In this case the interference pattern, localized in the hologram plane corresponds to the pattern presented in Fig. 6*b*.



*a* *b*

**Fig. 7.** Interference patterns.

In these two cases, as well as in others connected with variation of the radius of the wave front curvature of a spherical wave and its sign, the values of $G_3$ coefficients (which, in addition, can be determined by measuring $f$, $l_1$, $l_2$, and $R$) correspond to the values presented in Fig. 4 within 10% error of experimental measurements.

Let in the case of monitoring the diffuser longitudinal motion, the spatial filtering of the diffraction field be carried out in the hologram plane off of the optical axis, that is, assume, that the center of a filtering aperture has the coordinates $x_{03}$, 0. Then taking into account non-uniform motion of the subjective speckles, corresponding to the second exposure, and conditions that within the limits of the filtering aperture $d_f$ of the spatial filter $p_0$ (see Fig. 2), the phase change $kL_0^2\Delta l(x_3^2+y_3^2)/2l_1^2l_2^2$ does not exceed $\pi$ and $d_f \le 2\lambda l_1^2 l_2/d(l_1-L_0)\Delta l$, distribution of the complex field amplitude in the exit of the spatial filter takes the following form

$$u'(x_3,y_3) \sim p_0(x_3,y_3)\Bigg\{F(x_3+x_{03},y_3)\otimes \otimes\exp\left\{-\frac{iklL_0^2}{2l_1^2l_2^2}\left[(x_3+x_{03})^2+y_3^2\right]\right\}\otimes \otimes\exp\left(\frac{ikL_0x_{03}x_3}{l_1l_2}\right)P(x_3,y_3)+\exp(ik\Delta l)\times \times\left\{\int\int_{-\infty}^{\infty} t(x_1,y_1)\exp\left[\frac{-ikL_0(l_1-L_0)\Delta l x_1x_{03}}{l_1^3l_2}\right]\times \times\exp\left\{\frac{-ikL_0\left[x_1(x_3+x_{03})+y_1y_3\right]}{l_1l_2}\right\}dx_1dy_1\otimes \otimes\exp\left\{-\frac{ikl'L_0^2}{2l_1^2l_2^2}\left[(x_3+x_{03})^2+y_3^2\right]\right\}\otimes \otimes\exp\left(\frac{ikL_0x_{03}x_3}{l_1l_2}\right)P(x_3,y_3)\right\}\Bigg\}. \quad (27)$$

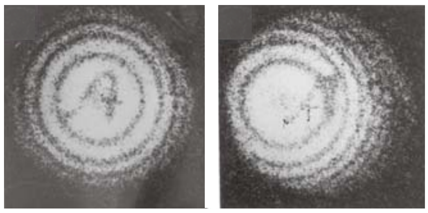
In the considered case, after making the Fourier transform, the distribution of the complex field amplitude in the plane $(x_4, y_4)$ (see Fig. 2) is determined by the following expression

$$u'(x_4,y_4) \sim \left\{p\left(x_4-\frac{L_0}{l_1}x_{03},y_4\right)t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\times \times\exp\left[\frac{ikl_1^2}{2lL_0^2}(x_4^2+y_4^2)\right]\exp\left(\frac{i2kx_{03}x_4}{l_2}\right)\times \times\left\{1+\exp(ik\Delta l)\exp\left[\frac{ikM\Delta l}{2l_1^2}(x_4^2+y_4^2)\right]\times \times\exp\left(\frac{ik(l_1-L_0)\Delta l x_{03}x_4}{l_1^2l_2}\right)\right\}\right\}\otimes P_0(x_4,y_4), \quad (28)$$

based on which light distribution in the recording plane *3* (see Fig. 2) takes the following form

$$I'(x_4,y_4)\sim \sim\left\{1+\cos\left[k\Delta l+\frac{kM\Delta l}{2l_1^2}(x_4^2+y_4^2)+\frac{k(l_1-L_0)}{l_1^2l_2}\Delta l x_{03}x_4\right]\right\}\times \times\Bigg|p\left(x_4-\frac{L_0}{l_1}x_{03},y_4\right)t\left(-\frac{l_1}{L_0}x_4,-\frac{l_1}{L_0}y_4\right)\times \times\exp\left[\frac{ikl_1^2}{2lL_0^2}(x_4^2+y_4^2)\right]\exp\left(\frac{i2kx_{03}x_4}{l_2}\right)\otimes P_0(x_4,y_4)\Bigg|^2. \quad (29)$$

As follows from comparison of Eqs. (19) and (29) a motion of interference fringes occurs, at moving the filtering aperture, with respect to the static image of the diffuser along the direction opposite to the motion of the lens $L$ pupil image (see Fig. 1). Thus, Fig. 8*a* corresponds to spatial filtering of the diffraction field on the optical axis, but Fig. 8*b* shows the same at a distance of 5 mm from the optical axis. In addition, dynamics of the interference pattern behavior is similar to the dynamics of its behavior in the case of the diffuser motion monitoring, when the center of a filtering aperture moves from the minimum of the interference pattern, localized in the hologram plane, to its maximum.



*a* *b*

**Fig. 8.** Interference patterns, localized in the Fourier plane and characterizing longitudinal motion of the diffuser, when spatial filtering is performed in the hologram plane: on the optical axis (*a*); at a distance of 5 mm from the optical axis (*b*).

Thus, the investigation results have shown the following.

The double-exposure recording with a negative lens of the quasi-Fourier holograms for monitoring of cross and longitudinal motions of a flat surface diffusely scattering incident light is accompanied by the formation of interference patterns, localized in two planes: in the hologram plane and in the far-field zone of the diffraction. For their recording, a spatial filtering of the diffraction field in the corresponding planes is necessary. For the interference pattern, localized in the Fourier plane and characterizing the diffuser cross motion, the interferometer sensitivity depends on both the magnitude and sign of the radius of curvature of a spherical wave front of a coherent radiation, used for illumination of the diffuser at the stage of the hologram recording. In its turn, in monitoring of the diffuser longitudinal motion, the interferometer sensitivity does not depend on the sign of the radius of curvature.

For the interference pattern localized in the hologram plane and characterizing cross or longitudinal motion of a flat surface diffusely scattering incident light, the interferometer sensitivity does not depend on the radius of curvature of a spherical wave front of a coherent radiation used at the stage of the hologram recording.

In the case of double-exposure recording of the Fourier hologram for monitoring of the diffuser cross motion, the interference pattern is localized only in the hologram plane. In its turn, recording of the interference pattern localized in the hologram plane and characterizing longitudinal motion of a flat surface diffusely scattering incident light, requires a spatial filtering of the diffraction field to be performed in the Fourier plane owing to the mismatch of the subjective speckles of the two exposures in the hologram plane.

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