MODEL ESTIMATIONS OF THE SCATTERING MATRIX OF A POLYDISPERSE ENSEMBLE OF ARBITRARILY ORIENTED CYLINDRICAL PARTICLES

R.F. Rakhimov and D.N. Romashov

*Institute of Atmospheric Optics,
Siberian Branch of the Academy of Sciences of the USSR, Tomsk*
Received February 28, 1991

We discuss a technique for modeling the aerosol light scattering parameters with an account of the nonsphericity and arbitrary orientation of particles. Some results of modeling the variability of the scattering matrix coefficients of a polydisperse ensemble of cylindrical particles of finite length depending on the angle of preferred orientation of particles are presented.

The specific character of formation of the phase composition of the high clouds assumes the existence of the crystal aerosol structures inside them and, as a consequence, the anisotropy of their optical characteristics. In this connection, a number of light scattering effects, which arise as a result of interaction of electromagnetic waves with cirrus, cannot be explained within the framework of model based on the Mie theory. The formulation of the problem on the variability of light scattering properties of the disperse phase outside the scope of the hypothesis of the spherical symmetry in the shape or dielectric characteristics leads to the sharp multiplication of the number of starting parameters of this problem. For example, an analysis of the optical–radar properties of cirrus in the cylindrical particle approximation calls for the study of the effect of not only the polydispersity (of two parameters: length and thickness) but also the orientation.

Based on the well–known generalization of the rigorous solution of the problem of light scattering by the infinite cylinder for the finite one (using the Huygens principle in the Fresnel formulation), we consider here some peculiarities of formation of the optical–radar characteristics of the disperse phase which is similar to cirrus in its microstructure.

In the Cartesian coordinate system with the z' axis along the axis of the cylinder of length l and radius r the relation for the amplitude function of the scattered electromagnetic wave has a form

$$S_i(\varphi', \theta', \beta, r, l) = \frac{kl}{\pi} E\left(\frac{kl}{2}(\cos\theta' - \cos\beta)\right) \times$$
$$\times T_i(\varphi', \beta, r)\ (i = 1, 2, 3, 4), \qquad (1)$$

where $T_i(\varphi', \beta, r)$ are the amplitude functions of the infinite cylinder, $k = 2\pi/\lambda$ is the wave number, β is the angle between the direction of wave incidence and the axis of the cylinder (it will here in after coincide with the second Euler's angle), φ' and θ' are the spherical coordinates, and $E(x) = \dfrac{\sin x}{x}$ is the Kotel'nikov function.

We shall specify the orientation of the cylinders in the coordinate system with the z axis along the direction of wave propagation by the two Euler's angles $0 \le \alpha \le 2\pi$ and $0 \le \beta \le \pi$.[5]

The relation between the spherical coordinates specified in the first coordinate system and the analogous coordinates in the second coordinate system is given by the formulas [5]

$$\cos\theta' - \cos\theta\cos\beta + \sin\theta\sin\beta\cos(\varphi - \alpha)\ ;$$
$$\text{cotan}\varphi' = \text{cotan}(\varphi - \alpha)\cos\beta - \text{cotan}\theta\sin\beta/\sin(\varphi - \alpha)\ . \qquad (2)$$

On account of Eqs. (2) we can write the relation for the amplitude functions of the field scattered by the arbitrarily oriented cylinder in a form

$$S_i(\theta, \varphi, \alpha, \beta, r, l) = \frac{kl}{\pi} \times$$
$$\times E\left(\frac{kl}{2}(\cos\theta\cos\beta + \sin\theta\sin\beta;\ \cos(\varphi - \alpha) - \cos\beta)\right) \times$$
$$\times T_i\left(\text{arccotan}\left(\text{cotan}(\varphi-\alpha)\cos\beta - \frac{\text{cotan}\theta\sin\beta}{\sin(\varphi-\alpha)}\right), \beta, r\right) \qquad (3)$$

For the polydisperse ensemble of finite cylinders with the length $g(l)$, radius $n(r)$, and two angles of orientation $q(a, b)$ distribution functions the coefficients of the scattering matrix, after the statistical averaging, can be written in a form

$$\overset{>}{S}_{ij}(\theta, \varphi) = \int_0^{2\pi}\int_0^{\pi}\int_{r_1}^{r_2}\int_{l_1}^{l_2} q(\alpha, \beta)\, n(r)\, g(l) \times$$
$$\times S_{ij}(\theta, \varphi, \alpha, \beta, r, l)\, d\alpha\, d\beta\, dr\, dl\ . \qquad (4)$$

For the model estimations let us consider only the optical properties of the polydisperse ensemble with azimuth symmetry in the distribution of the axes of the cylinders

$$q(\alpha, \beta) = f(\beta)/2\pi\ . \qquad (5)$$

For such ensembles the scattering matrix $\hat{S}_{ij}(\theta, \varphi)$ is independent of the angle φ, i. e.

$$\hat{S}_{ij}(\theta, \varphi_1) = \hat{S}_{ij}(\theta, \varphi_2)\ . \qquad (6)$$

0235-6880/91/07 496-05 $02.00

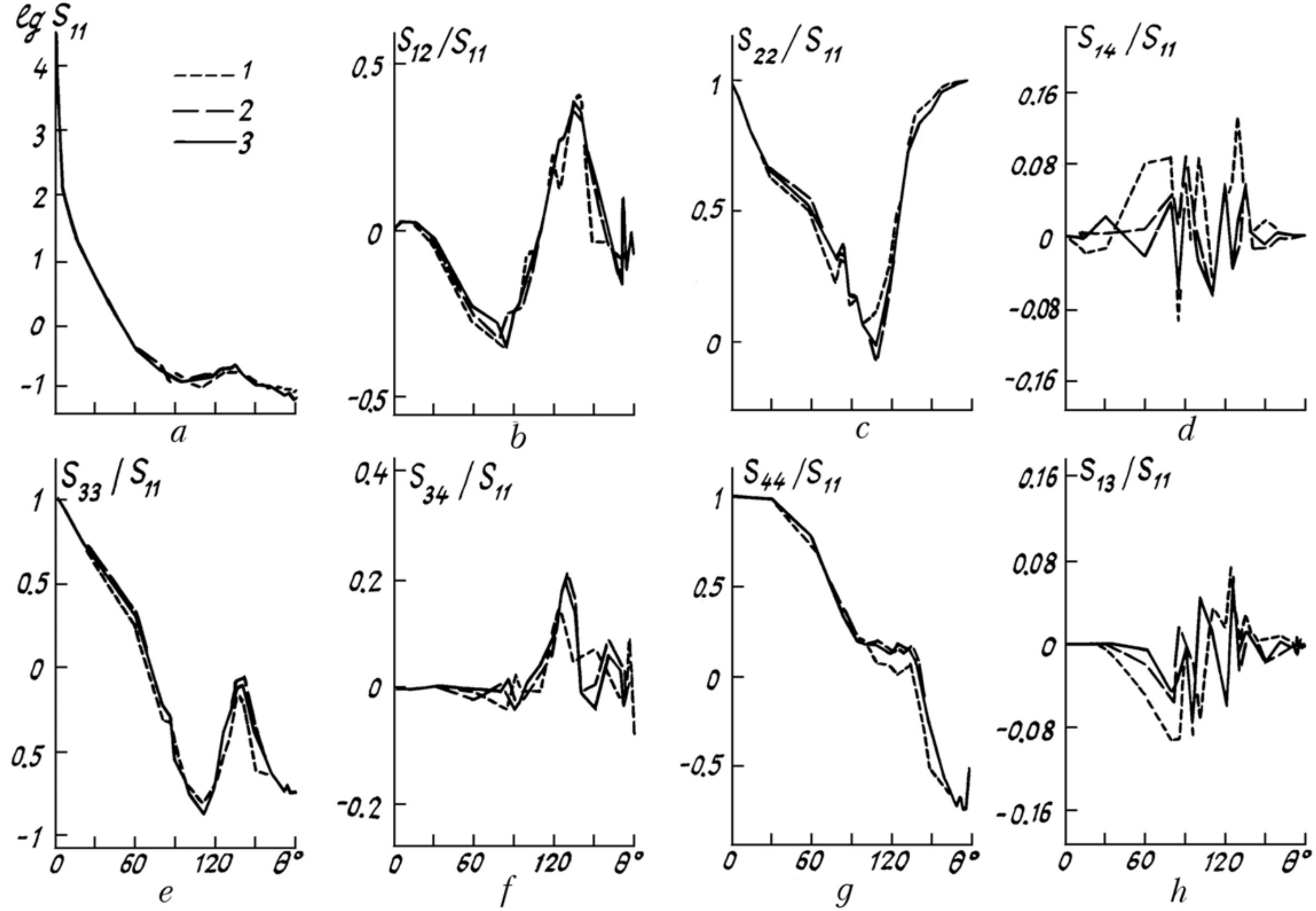

FIG. 1 Normalized coefficients of the scattering matrix of the cylindrical ice particles as functions of the quantity of the statistically averaged data N_c: *1)* $1.5\cdot10^4$; *2)* $3.0\cdot10^4$; *and, 3)* $4.5\cdot10^4$ realizations.

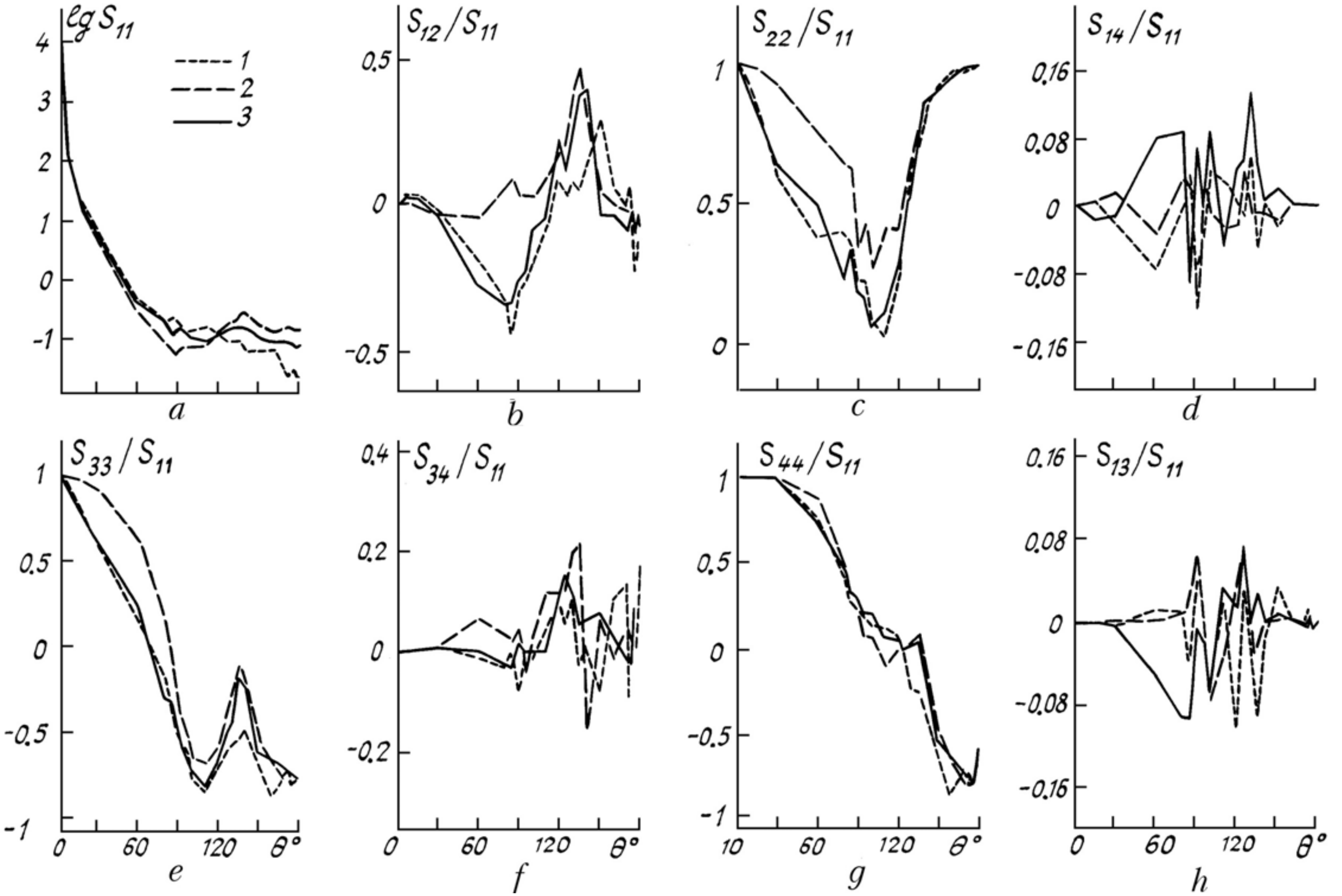

FIG. 2. Normalized coefficients of the scattering matrix of the ice particles vs the preferred slope angle of the axes of the cylinders with respect to the direction of wave incidence for r_m = 1.0 μm *at* λ = 0.6943 μm: *1) normal distribution with* $\beta_m = \pi/4$; *2) normal distribution with* $\beta_m = \pi/2$; *and, 3) uniform (chaotic) distribution* $q(\beta)$.

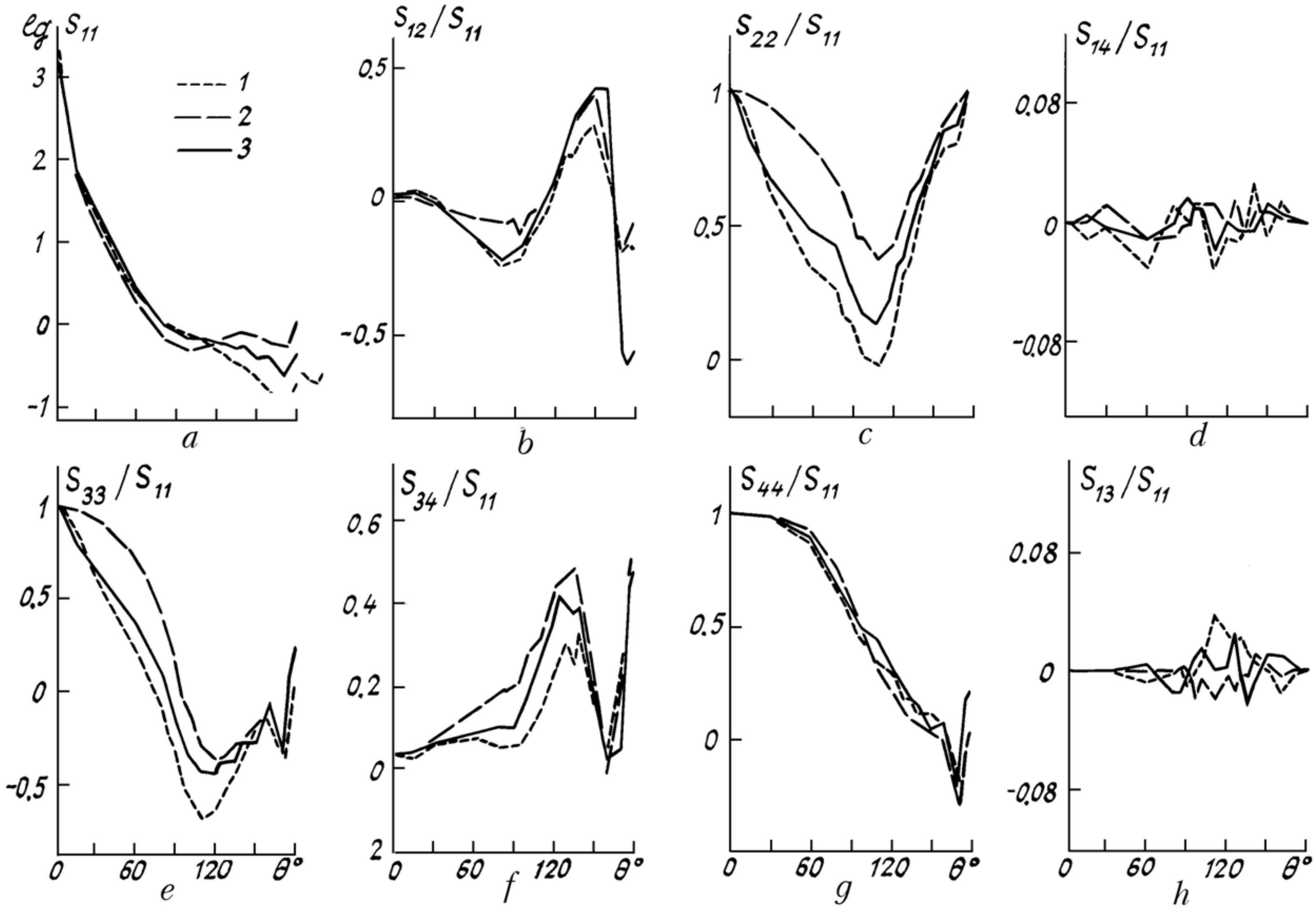

FIG. 3. The same as in Fig. 2 but for $r_m = 10.0$ μm *at* $\lambda = 1.06$ μm.

The evaluation of multiple integral (4) using the cubature formulas requires large expenditures of computing time, therefore, we made the statistical averaging of the optical characteristics with the help of the Monte Carlo method. This method makes it possible to expand the typological variety of the analyzed states of the disperse phase and to consider not only the situations with chaotic orientation of the particles,[4] i.e., with uniform distribution over the directions of particle orientations.

The problem of convergence of the employed method was studied on the basis of the numerical estimates. The results of calculations show that with increase of the characteristic particle size and expansion of the range of deviation of the angle defining the orientation of the axis of the cylinder from the preferred orientation angle the quantity of data in statistical averaging N_c, which is necessary for the stability of the obtained estimates, can increase from 15 up to 45 thousands of realizations.

Figure 1, *a–h* show the phase functions of aerosol light scattering calculated for the polydisperse ensemble of the cylindrical particles obeying the lognormal distribution over the radii of cross sections, uniform (in the range from 3 to 5) with respect to the elongation factor $j = l/r$ and normal with respect to the orientation angle β. For all the coefficients of the scattering matrix with increase of the quantity of data in statistical averaging the stabilization of the shape of the angular dependences is typical. This stabilization is sufficient for the determination of the fine structure such as, the local maximum at $\theta \sim 170–180°$ (Fig. 1 *a*, *b*, *e*, and *f*). As for the oscillations in $S_{13}(\theta)$ and $S_{14}(\theta)$, they are caused by their small absolute values due to the chaotic orientation of the cylinders in relation to the azimuth angle φ, and by the calculational error due to the limited word size of the computer. For example, the ratios $S_{14}(\theta)/S_{11}(\theta)$ calculated for the individual orientation of the cylinder vary from –0.8 to 0.8 and are compensated by summing over the contributions of the chaotically oriented cylinders. The angular dependence is described by a random function in the vicinity of zero due to the finite and discrete character of the data statistics.

The transformation of light scattering phase functions vs microphysical parameters of the disperse ensemble of arbitrary oriented particles was calculated based on the proposed technique.

Figure 2 *a–h* show the calculated data illustrating the transformation of the coefficients of the scattering matrix depending on the peculiarities in the orientation of the particles obeying the lognormal size distribution in the vicinity of $r_m = 1.0$ μm and $\sigma = 0.5$. Solid curves correspond to the chaotic orientation of the axes of the cylinders in relation to the angles α and β . Dash–dot and dashed curves have been calculated for the particles obeying the normal distribution of the axes of the cylinders with the average slope angle $\beta_m = 45$ and 90°, respectively. The standard deviation was set $\sigma = 30°$ in both cases. Analogous data are shown in Fig. 3 *a–h* for larger cylinders with $r_m = 10.0$ μm.

The same tendency of transformation of the light scattering phase functions can be seen independently of the particle size and dielectric constants (calculations were made at the wavelengths $\lambda = 0.6943$ μm with the refractive index of ice $m = 1.308–0.0001i$ and at $\lambda = 1.06$ μm with $m = 1.296–0.0001i$). The calculated results show that if for the spherical particles the phase functions of the coefficients the scattering matrix $S_{22}(\theta)$ and $S_{11}(\theta)$ are identical (see Eq. (4.77) in Ref. 1) then for cylindrical particles (depending on the orientation) the behavior of these functions differ markedly practically at all the scattering angles, except for the forward ($\theta = 0°$) and backward ($\theta = 180°$) directions, particularly at $\theta = 45°$.

It should be noted that for large particles (Fig. 3, *a*, *b*, and *e*) the peaks can be seen at the scattering angles θ varying from 125° to 140° which is referred to as the rainbow range for water–droplet clouds. The efficiency of radar scattering depends strongly on the angle of preferred orientation of particles. The backscattering phase function decreases with decrease of the slope angle of the axis of the cylinder with respect to the incident wave direction.

The angular dependences of the ratios $S_{33}(\theta)/S_{11}(\theta)$ and $S_{44}(\theta)/S_{11}(\theta)$, which are identical for the spherical particles and at the scattering angle $\theta = \pi$ are equal to –1, are different. In our case these ratios are not identical, different from –1, and vary depending on the characteristic particle size and the preferred orientation angle. Analogous differences from the spherical particles can be seen for radar scattering angles in the ratios $S_{12}(\pi)/S_{11}(\pi)$ (the degree of polarization) and $S_{34}(\pi)/S_{11}(\pi)$. Two last results are fundamental and indicate the necessity of detailed study, since they demonstrate the characteristic features of nonsphericity of the light scattering particles, and, as Figs. 2 and 3 show, depend strongly on their size.

REFERENCES

1. C.F. Bohren and D.R. Huffman. *Absorption and Scattering of Light by Small Particles* (Interscience – Wiley, New York, 1986).
2. Kuo–Nan–Liou. Appl. Opt. **11**, No. 3, 667–674 (1972).
3. G.N. Glasov, G.M. Krekov, and R.F. Rakhimov, in: *Abstracts of Reports at the 2 nd All–Union Symposium. on the Propagation of Laser radiation in the Atmosphere*, Tomsk, (1973)
4. O.A. Volkovitskii, L.N. Pavlova, and A.G. Petrushin, *Optical Properties of Crystal Clouds* (Gidrometeoizdat, Leningrad, 1984) .
5. D.A. Varshalovich, A.N. Moskalev, and V.K. Khersonskii, *Quantum Theory of the Angular Moment* (Nauka, Leningrad, 1975).