TECHNIQUE FOR CORRECTING SPECTRAL BEHAVIOR OF THE AEROSOL OPTICAL DEPTH OF THE ATMOSPHERE

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This paper presents a statistical approach to correcting the spectral behavior of the aerosol optical depth of the atmosphere and eliminating systematic errors in this component of the atmospheric extinction. This method can be useful in studies of aerosol optical depths of the atmosphere along horizontal and slant paths as well as in the laboratory studies.

In experimental studies of aerosol optical depth of the atmosphere in a wide wavelength range along horizontal and slant paths there arises the problem associated with the correction of their values and spectral behavior. This is accounted for by use of "residual principle" when inferring aerosol optical depth τ determined as difference between the measured total optical thickness ε and simulated calculations of optical thicknesses of molecular components of extinction β. Used "residual principle" results in the dependence of the calculation error τ on accuracy characteristics of β and ε.

Different techniques for taking account of β yield different values of τ. Thus, e.g., empirical formulas for calculating continuous absorption by water vapor within 8–13 μm range (Refs. 1 and 2) provide the values of extinction which noticeably differ for large absolute humidities of air.

The measurement accuracy of ε implies a systematic error caused by inaccurate determination of value of "zero signal", i.e. intensity of radiation I_0 before its propagation along the measurement path which enters into Bouguer's law

$$I = I_0 \, e^{-\varepsilon} .$$

A "long" Bouguer's law used for slant atmospheric paths can lead to over– or underestimating "zero signal" depending on type of diurnal behavior of ε. Thus, e.g., the calculations[3] revealed that the optical depths in the regions of Vladivostok and cape Karadag were underestimated 35 and 28% in magnitude, respectively, due to maximum appearing in the diurnal behavior of ε. The "residual principle" results in transition of a systematic error from ε to τ. Hence the aforementioned factors significantly distort the spectral behavior and the values of τ. Therefore a demand arose for a method which could eliminate disadvantages of the "residual principle" for separating out the aerosol optical depth. This method is described in present paper.

Correction is made based on a calibrating data subarray incorporating either all of the experimental data or some amount of them. It must satisfy the following requirements.

1. The neighboring wavelengths with the experimental values λ must be close to each other for the correction of τ to be presice. The criterion of closeness is described below.

2. A linear relation and significant correlation mast occur among aerosol optical depths at neighboring wavelengths. The linear relation is tested by the method of least squares.[4] If for large values of τ there appears a nonlinearity, then these points must be excluded from the subarray under consideration.

3. In the calibrating subarray there mast be τ which are equal to zero or differ from zero by no more than a value of a single measurement error over the entire spectral range studied. The fulfilment of this requirement is varified after correction of τ. This condition should be fulfilled to be certain that the relation between aerosol optical depth is linear for their values being close to zero.

To find the relation between aerosol optical depths at neighboring wavelengths the equation of linear orthogonal rms regression is used[4]

$$\tau_i = K_{0\,i} + K_{i,\,i+1} \, \tau_{i+1} , \tag{1}$$

where $i = 1, 2, ..., n - 1$, n is a number of investigated spectral intervals. In this case the regression coefficients $K_{i,\,i+1}$ are determined by the formula

$$K_{i,i+1} = \frac{1}{2\,\rho_{i,\,i+1}} \left(\frac{\sigma_i}{\sigma_{i+1}} - \frac{\sigma_{i+1}}{\sigma_i} \right) \pm$$

$$\pm \sqrt{ \frac{1}{4\,\rho_{i,\,i+1}^2} \left(\frac{\sigma_i}{\sigma_{i+1}} - \frac{\sigma_{i+1}}{\sigma_i} \right)^2 + 1 } , \tag{2}$$

where $\rho_{i,\,i+1}$ are the normalized coefficients of correlation between τ_i and τ_{i+1}; while σ_i and σ_{i+1} are their rms deviations. A sign of the expression before the square root in Eq. (2) is correspond to that of the correlation coefficient $\rho_{i,\,i+1}$.

The equation of orthogonal rms regression provides physically true coefficients $K_{i,\,i+1}$. To check this, let us assume that there are two arrays τ_i at a single wavelength. Then their rms deviations are equal and the correlation coefficient is a function of a random error of single measurement of τ. Using a conventional equation of rms regression we obtain the following coefficient $K_{i,\,i}$:

$$K_{i,\,i} = \frac{\sigma_i}{\sigma_i} \, \rho_{i,\,i} , \tag{3}$$

where $\rho_{i,\,i}$ is the coefficient of correlation between aerosol optical depths at a single wavelength. As seen from Eq. (3), the regression coefficient $K_{i,\,i}$ equals unity only for $\rho_{i,\,i} = 1$, through it follows from the setting up a problem that this equality must be fulfilled for any values of $\rho_{i,\,i}$ ($\rho_{i,\,i} > \rho_0$,

where ρ_0 is the level of significant correlation). When the equation of orthogonal rms regression is used, the coefficient $K_{i,\,i}$ equals unity with any value of $\rho_{i,\,i}$. Hence this equation gives a physically correct result. By virtue of the requirement on closeness of neighboring wavelengths which implies closeness of a random error of reconstructing τ_i based on the values of τ_{i+1}

$$\delta_i = \sigma_i \sqrt{1 - \rho_{i,\,i+1}^2} \sqrt{\frac{n-1}{n-2}}, \qquad (4)$$

to a random error of reconstructing τ_i based on its own values

$$\delta_{0i} = \sigma_i \sqrt{1 - \rho_{i,\,i}^2} \sqrt{\frac{n-1}{n-2}}, \qquad (5)$$

we obtain correct values of coefficients $K_{i,\,i+1}$. As seen from Eqs. (4) and (5) the requirement on closeness of wavelengths is reduced to closeness of the correlation coefficients $\rho_{i,\,i+1}$ and $\rho_{i,\,i}$. The physically true values $K_{i,\,i+1}$ enable one to set the coefficients $K_{0\,i}$ equal to zero. Physically this means that when there is no aerosol extinction at one wavelength (i.e., when concentration of an optically active aerosol equals zero), there is also no aerosol extinction at the other wavelength which is close to the first one. From this it follows that if in Eq. (1) the constants $K_{0\,i}$ for a calibrating subarray are nonzero, then the spectral behavior of τ must be corrected. Clearly the mean corrected values of aerosol optical depths $(\overline{\tau}')$ are related to each other via the relation

$$\overline{\tau}'_1 = K_{12}\,\overline{\tau}'_2 = K_{12}\,K_{23}\,\overline{\tau}'_3 = \dots = K_{12}\,K_{23}\,\dots\,K_{n-1,\,n}\,\overline{\tau}'_n, \quad (6)$$

and the mean corrected and uncorrected values τ are related to each other via the relation

$$\overline{\tau}'_i = \overline{\tau}_i - \Delta_{0\,i}, \qquad (7)$$

where $\Delta_{0\,i}$ is the systematic error at the ith wavelength $(i = 1, \dots, n)$ which consists of Δ_0 and Δ_i. The value Δ_0 is constant for all of the spectral intervals and caused by common factor, such as inaccurate determination of the lengths of the measuring (long) and calibrating (short) atmospheric paths. The value Δ_i, having different absolute values, is a sign−changing value and caused by a random error in signal measurement along the calibrating path.

When considering the problem on correction of aerosol optical depths we can separate out three approaches. The first approach allows one to find and eliminate systematic errors Δ_0 and Δ_i $(i = 1, \dots, n)$ in τ and the second and third ones make it possible to correct the spectral behavior of τ.

The first approach assumes obtaining "zero signals" or their relative spectral behavior at several wavelengths. To this end it is desirable to use such spectral intervals, where extinction is determined only by aerosol (without Rayleigh scattering) or molecular component can be taken into account with reasonable accuracy. In the remaining segments of the spectrum the "zero signals" are taken randomly and the technique for finding corrections at these wavelengths is described below.

The values of systematic errors Δ_0 and Δ_i are found from the conditions of minimum of sum

$$\Psi = \sum_{i=1}^{n} \left(K_i - \frac{\overline{\tau}_i - \Delta_0}{\overline{\tau}_1 - \Delta_0 - \Delta_1} \right)^2 \qquad (8)$$

and equality to zero

$$\sum_{i=1}^{n} \left(K_i - \frac{\overline{\tau}_i - \Delta_0}{\overline{\tau}_1 - D_0 - \Delta_1} \right)^2 = 0, \qquad (9)$$

where the coefficients $K_i = \overline{\tau}'_i / \overline{\tau}'_1 = 1 / \prod\limits_{j=1}^{i-1} K_{j,\,j+1}$ represent a relative spectral behavior of mean values of aerosol optical depths for the calibrating subarray. From Eq. (9) we obtain

$$\Delta_1 = \overline{\tau}_1 - \Delta_0 - \frac{\sum\limits_{i=1}^{n} \overline{\tau}_i - n\,\Delta_0}{\sum\limits_{i=1}^{n} K_i}. \qquad (10)$$

Substituting Eq. (10) into Eq. (8) and taking the first derivative of the sum Ψ with respect to Δ_0 equal to zero we obtain the expression for Δ_0

$$\Delta_0 = \frac{\sum\limits_{i=1}^{n} \overline{\tau}_i \sum\limits_{i=1}^{n} K_i\,\overline{\tau}_i - \sum\limits_{i=1}^{n} K_i \sum\limits_{i=1}^{n} \overline{\tau}_i^2}{n \sum\limits_{i=1}^{n} K_i\,\overline{\tau}_i - \sum\limits_{i=1}^{n} K_i \sum\limits_{i=1}^{n} \overline{\tau}_i}. \qquad (11)$$

The remaining errors Δ_i $(i = 2, \dots, n)$ are found from the expression

$$\Delta_i = \overline{\tau}_i - \Delta_0 - K_i\,(\overline{\tau}_1 - \Delta_0 - \Delta_1). \qquad (12)$$

The systematic errors Δ_0 and Δ_i can also be determined by equating the partial derivatives of the sum (8) with respect to Δ_0 and Δ_i to zero. It is easy to show that these two techniques for obtaining Δ_0 and Δ_i yield equal results.

The second approach is based on the use of τ at a single jth wavelength as a reference one and allows one to find a "zero signal" only in this spectral interval. In the remaining spectral intervals "zero signals" are chosen arbitrary and obtaining values of τ are corrected.

The corrections to aerosol optical depth are obtained from the expression

$$\Delta\tau_i = \overline{\tau}_i - (K_i / K_j)\,\overline{\tau}_j. \qquad (13)$$

The absolute systematic errors introduced with this approach to optical depths at the ith wavelength are

$$\Delta_{0\,i} = (K_i / K_j)\,(\Delta_0 + \Delta_j), \qquad (14)$$

and the relative ones are

$$\Delta_{0\,i} / \overline{\tau}'_i = (\Delta_0 + \Delta_j) / \overline{\tau}'_1\,K_j. \qquad (15)$$

Clearly, a spectral interval where a relative error is minimum must be chosen as a reference one. If the sums $\Delta_0 + \Delta_j$ $(j = 1, \dots, n)$ are close to each other, what is usually observed under real atmospheric conditions, then the minimum values of expression (15) are derived at maximum coefficient K_j or maximum aerosol optical depth

τ_i'. Then in measuring of τ in the visible spectral range a short–wave segment of the spectrum must be taken as a reference one.

The third approach is based on the use of the method of "minimal points" in the long–wave spectral range, where extinction is the weakest. The method of "minimal points" assumes that the minimum value of aerosol optical depth is taken equal to zero.

The procedure of determining the absolute values of τ is as follows. First "zero signals" are taken arbitrary for all of the spectral intervals, then τ at any wavelength is taken as a reference one, and the corrected aerosol optical depths are found based on the second approach described above. In the subarray obtained with this procedure there are several smallest values of τ, they are varified and the minimal one is chosen. Let it be τ_{0j}. Then the corrected mean value of τ at the jth wavelength is

$$\overline{\tau}_j' = \overline{\tau}_j - \tau_{0\,j} \ . \tag{16}$$

Corrections to τ at the other wavelengths are found from the expression

$$\Delta\tau_i = \overline{\tau}_i - (K_i/K_j)\,(\overline{\tau}_j - \tau_{0\,j}) \ , \tag{17}$$

and the running corrected values of aerosol optical depths are

$$\tau_i' = \tau_i - \overline{\tau}_i + (K_i/K_j)\,(\overline{\tau}_j - t_{0j}) \ . \tag{18}$$

The obtained values represent the lower estimate of τ. It should be noted that this method requires sufficiently long series of observations τ, since in this case the appearance of values τ close to zero in the spectral range under study becomes more probable.

Thus, three approaches to the problem of correcting the spectral behavior of aerosol optical depth are considered in this paper. In the first one the correction is accomplished by removing systematic errors in τ, in the second and third approaches it is done by introducing corrections to aerosol optical depths.

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