Reconstruction of signals at space detection and ranging of optical radiation sources through cloud layers

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The algorithm for reconstruction of a signal is proposed to be employed in the space information-measuring system (SIMS) for detection and ranging of a source of pulsed optical radiation from space through a cloud layer. The analytical model of a path of signal propagation through the atmosphere with a cloud layer is used for deconvolution of signals. A modification of the well known mathematical apparatus, without changing characteristics of the model, as applied to direct problems, allows solution of the inverse problem, namely, determination of the values of the path's parameters from SIMS data and, finally, deconvolution of the signal transmitted by a source. Computations are made automatically with a digital processor equipped with a fast transversal filter in the additive operating mode.

1. Space global observation systems (SGOSs) including spacecraft, ground-based and space-borne computer systems, and equipment for remote sensing form a class of hierarchical information systems, whose elements are related to each other by the channels for exchange of information – symbolic messages. The measuring tools enter into the SGOS as sources (not the only ones) of information.[1] The presence of measuring tools alone does not yet make a SGOS a measuring system.

Among possible applications of a SGOS as a measuring system there are detection, ranging, and measuring of characteristics of light emitting objects from their emissions. The case in point deals with indirect measurements; problems of optical remote sensing[2,3] fall in the category of inverse and, as a rule, ill-posed problems.[3,4] Methods of simulation of signal fields at inputs of space-borne receiving systems play an important part in solution of such problems.[5] The SGOS becomes a specialized space-borne information-measuring system (SIMS), if it includes adequate models of the objects sensed, signal propagation path, and the background, as well as the corresponding algorithms for data processing. These elements are an integral part of SIMS.

The SIMS can be considered as a hypothetical measuring device (a version of the measuring and computational system), whose limiting capabilities can significantly exceed those of the initial one. The methods of reduction to such a hypothetical device in the Hilbert space have been developed in Ref. 6 as applied to the simplest mathematical model of measurements $[A, \Sigma]$, given by the pair of operators (A is the model of a measuring device, Σ is the correlation operator of measurement errors), with and without restrictions on the noise level. In the considered case of detection and ranging of radiation sources, the problem of reduction becomes more complicated: additional difficulties associated with signal propagation through the atmosphere and signal deconvolution may arise.[7] This paper is devoted to analysis of these difficulties and the ways to overcome them.

The Monte Carlo method when applied to statistical simulation of the effects of radiation interaction with medium scatterers, in particular, fluctuations of the refractive index (Rayleigh scattering), aerosol particles, and water droplets of rain and clouds (Mie scattering), provides the best estimates of the influence of atmosphere on the signal propagating through it. Capabilities of the Monte Carlo method in application to problems of atmospheric optics are well known. This method is usually used as a reference one. However, its drawback is that it requires too cumbersome calculations. Among the numerical methods, only the Sivert F_n-method based on expansion of the solution over the complete system of Keiz eigenfunctions (singular eigenfunctions of the homogeneous transfer equation) compares with the Monte Carlo method in accuracy and reliability. However, this method is cumbersome too.[8]

Application of statistical simulation to monitoring of situations on the Earth's surface is restricted to checking of results obtained with other, more efficient, computational methods.

Computational methods that are based on simple analytical models of the propagation path involving only few parameters are quite efficient. Such a model has earlier been proposed in Ref. 9. It well describes the signal propagation through optically thick atmospheric layers. However, within the framework of this model it is difficult to solve inverse problems associated with determination of the parameters, which characterize the path from the system output data. At the same time, it is not always possible to determine the path parameters using direct methods, especially, on a real time scale. Below we describe a modified version of the theory based on the assumptions of the same degree of generality as in Ref. 9, but having the form which is more convenient for solving problems of remote sensing.

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2. The consideration is carried out within the framework of the theory of systems and transformations with the use of mathematical apparatus of Fourier optics. The path of signal propagation is considered as a linear system, invariant to shifts. By the signal, with respect to which the system characteristics are defined, is meant the envelope of the pulse shape emitted by a radiation source. The frequency band, within which the transfer function of the path for such signals is non-zero, does not exceed 100 kHz. It corresponds to the radio frequency region. Optical radiation is only a carrier of the signal; its frequency plays the role of a carrier frequency.

The case in point is the reconstruction of the initial signal $f(t)$ emitted by the source. When recorded, this signal is presented by the convolution with the pulse response of the path $h_i(t)$:

$$f_i(t) = f(t)*h_i(t) + n_i(t),\ i = 1, \ldots, m \qquad (1)$$

or, in the frequency region,

$$F_i(\omega) = F(\omega)\, H_i(\omega) + N_i(\omega),\ i = 1, \ldots, m. \qquad (2)$$

Here m is the number of space platforms that have recorded the signal; usually $m = 5$–8. When writing Eqs. (1) and (2), we simplify the problem, restricting ourselves to consideration of the pulse response $h_i(t)$ and the transfer function $H_i(\omega)$ of the path and omitting, for simplicity, the instrumental function of the receiving device $h_{\text{ins}}(t)$ and, correspondingly, $H_{\text{ins}}(\omega)$. The signal is recorded against the background of noise. The noise is additive with the Gaussian distribution in time (n) and frequency (N) regions. The structure of response is known; it depends on the model of the path. However, values of parameters entering into the model are not always known. They should be found from the recorded signals $f_i(t)$ distorted by the noise.

The problem of reconstructing the function $f(t)$ from the recorded data $f_i(t)$ is an ill-posed one; it is presented by the integral equation (1) of the first kind. The following factors are used for its regularization: excessiveness of the system of equations (2), assumption of the frequency boundedness of all functions involved in calculations, smoothing properties of the additive NK-filter used for estimation of the model parameters.

The model of the path is constructed in the following way. First we determine the intensity distribution and time lags of the radiation wave (integral lag) along all ray trajectories from A to B at the upper boundary S_2 of a cloud layer (Fig. 1) under the harmonic action $\exp(i\omega t)$ at the input of the path. Then we sum up, at the receiving aperture of a spacecraft, all partial waves corresponding to different trajectories of signal propagation through the cloud layer.

It is more convenient to integrate over the surface S_3 rather than S_2. The surface S_3 is normal to the ray $n\,B$; its trace is shown in Fig. 1 by the line DE. For an "infinitelyB far ($n\,B \approx 2 \times 10^4$ km) receiving device, the bunch of rays from the emitting spot on S_3 (15 to 20 km in diameter) can be considered as a plane parallel, neglecting the difference between the angle θ and, for example, θ_1.

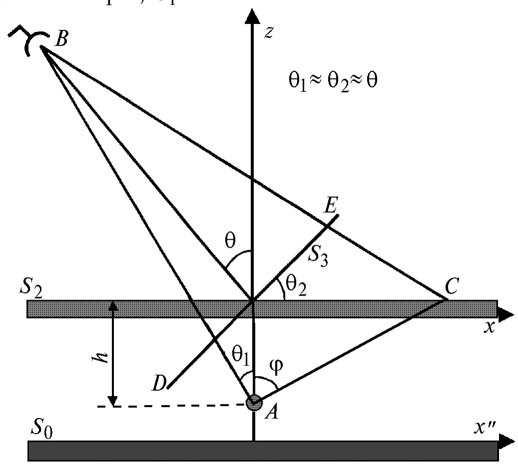

Fig. 1.

The spatiotemporal distribution of the intensity of optical radiation on S_3 can be presented as

$$I(r, \psi; \omega) = f(r, \psi)\, f(\Delta_r) \exp[-i\,\omega\tau(r, \psi; t_0) - i\,\omega\,\Delta_r]. \quad (3)$$

Parameterization in S_3 is done with the use of the system of rays: the variables r and ψ characterize the point on the S_3 that corresponds to the ray passing through the surface S_3 at the point with cylindrical coordinates r and ψ. The designation $\tau = \tau_0 + \tau_1$ is used for the delay τ_0 of the wave at the point r in comparison with the wave propagating along the straight trajectory AB, $\tau_0 = t_0\,[(\cos\varphi)^{-1} + x \sin\theta - \cos\theta]$, as well as the total delay caused by the diffuse character of radiation in the layer $\tau_1 = t_0\,KR \approx 0.73(H/h)\, t_0\, R$ (Ref. 10). Upon introducing the variable $R = 1/\cos\varphi = \sqrt{1 + (r/h)^2}$, let us present τ in the form

$$\tau = t_0\,[R(1 + K) + \sqrt{R^2 - 1}\,\sin\theta\,\cos\psi - \cos\theta]. \quad (4)$$

The structure of this equation has the greatest effect on the transfer function of the system $H(\omega)$.

The functions $f(r, \psi) = f(r)$ and $f(\Delta)$ describe the radiation flux density at the point (r, ψ) and the spread of photons' mean free paths (delay times) for the rays arriving at this point from the source.

To calculate $f(r)$, we should first solve the problem of transfer of unpolarized optical radiation through the cloud layer to a spacecraft from a point source separated by $(h - H)$ distance from the cloud bottom. This problem can hardly be solved directly. Let us use the approach proposed in Ref. 11. The plane wave with the intensity $\sim(hR)^2$ is thought to be incident on every elementary area of the cloud bottom at an angle $\varphi = \arccos\,(1/R)$. Propagation of a plane wave through the layer is calculated with the approximate methods of transfer, which allow simple analytical solutions for optically thick layers. In our calculations we used Chou method,[12] which realizes Hartel idea on presentation of the radiation field in a layer as a sum of intensities of different scattering orders, each being characterized by its own phase and weighting functions. Besides, the calculations have been done, in parallel, by the method of approximation of Ambartsumyan equations. Both of these methods gave close results, which can be well approximated by the smoothing function

$$f(r) = CR^{-1} \exp(-R/R_0), \quad R_0 = 2/3 . \qquad (5)$$

The value $R_0 = 2/3$ can be corrected (if necessary) in the experiment on determination of the path's parameters.

The spread of the photons' mean free paths for each trajectory is described by the normalized exponential function $f(\Delta_r)$ with the distribution width $\alpha\, t_0 \approx 2R(H/c) \approx 3H/c$. This is somewhat different from that accepted earlier in Ref. 9, but agrees well with the data from Ref. 13 and practically does not change the form of the function $H(\omega)$.

3. Upon integration of the intensity distribution on the surface S_3 over the variables r, ψ, and Δ_r, taking into account the equality $r\mathrm{d}r = h^2 R\mathrm{d}R$ and Bessel equation

$$(2\pi)^{-1} \int_0^{2\pi} \mathrm{d}\psi \exp(iv\cos\psi) = J_0(v),$$

we obtain the following analytical equation:

$$H(\omega) = A_0[(i\omega\alpha t_0 + 1)^{-1}\cos\theta]\exp(i\,\omega t_0\cos\theta) \times$$
$$\times \int_1^{\infty} J_0(\omega t_0 \sin\theta\sqrt{R^2-1})\exp(-p^R)\mathrm{d}R ; \qquad (6)$$
$$p = (R_0)^{-1} + i\,\omega t_0\,(1+K),$$

which is convenient for making calculations. Integrals of this type are known quite well; they occur in different physical problems, for example, when calculating infinitely long lines with loss but without leakage.[14] Taking into account the formulas of Laplace transformation, we derive from Eq. (6) the transfer function of the path:

$$H(\omega) = A_0[(i\omega t_0 + 1)^{-1}\cos\theta]\exp(i\omega t_0\cos\theta)\,W(\omega) ; \quad (7)$$
$$W(\omega) = \{\exp[-s\sqrt{(i\omega + r)^2 - q^2}]\}[s\sqrt{(i\omega + r)^2 - q^2}]^{-1},$$

where

$$s = t_0(1+K)\sqrt{1-(\sin\theta)^2(1+K)^{-2}} ; \qquad (7a)$$
$$r = (1+K)(s^2 R_0)^{-1} ; \quad q = (1+K)^{-1} r\sin\theta .$$

For the pulse response of the system $h(t)$ we have in this case:

$$h(t) \triangleq F^{-1}\{H(\omega)\} = h_1(t) * h_0(t) , \qquad (8)$$

where

$$h_1(t) = (\alpha\, t_0)^{-1}\exp[-t(\alpha\, t_0)^{-1}] ,$$
$$h_0 = A_0(\alpha\, t_0)^{-1}\exp(-r\,t_0\cos\theta)\,s^{-1}\exp(-r\,t) \times$$
$$\times I_0(q\sqrt{(t+t_0\cos\theta)^2 - s^2})\,U(t-\delta_1) . \qquad (8a)$$

Here $\delta_1 = s - t_0\cos\theta$ is the delay caused by signal propagation through the cloud layer.

The constant A_0 in Eqs. (7) and (8) can be found from the condition of energy balance:

$$H(0) = \int_1^{\infty} h(t)\,\mathrm{d}t = E = \eta\Omega(\pi)^{-1} E_0\cos\theta ,$$

where e is the power recorded at the output of the system assuming a delta-pulse at the system input; e_0 is the power emitted by the source into the upper half-space (in our case $e = 1/2$); η is the energy transmission coefficient of the cloud layer (0.2 to 0.3 at the optical thickness of the layer $\tilde{\tau} \ge 20$); $\Omega = Sz^{-2}$ is the solid angle of the signal reception (S is the area of the entrance pupil, z is the distance from the source to the receiver). From $H(0) = A_0\cos\theta W(0)$ it follows that

$$A_0 = (R_0)^{-1}\exp(1/R_0)\,\eta\Omega(2\pi)^{-1} .$$

4. Besides the observation angle θ, the variable parameters of the model are the altitude of the cloud top h and the geometrical thickness of the cloud layer H. They enter into Eqs. (7)–(10) through $t_0 = h/c$, $K = 0.73(H/h)$, and $\alpha = 3H/h$. Below we consider the possibility of determining these parameters from the data of measurements carried out with SIMS in the process of detection and ranging of a source. The experimental data can be also used for simultaneous refinement of the constant R_0, which characterizes the size of the emitting spot in the cloud at a given h, as well as for checking the correctness of estimating the observation angles θ. Equations (7) and (8) do not explicitly include the optical thickness of the layer $\tilde{\tau}$. The only requirement is that it is sufficiently large for making feasible the description of propagation processes in a cloud layer by the transfer equation in the diffusion approximation ($\tilde{\tau} > 15$).

In the process of calculation, it is important to know the ratio of $F_i(\omega) \equiv F(\omega; \theta_i)$ values at different angles θ:

$$\xi_{ij}(\omega) = F_i(\omega)/F_j(\omega) = H_i(\omega)/H_j(\omega) . \qquad (9)$$

For the angles $\theta_i = \theta_1$ and $\theta_j = \theta_2$ we have

$$\xi_{12}(\omega) = (\cos\theta_1/\cos\theta_2)\,(u_1/u_2) \times$$
$$\times \exp[u_2 - u_1 + i\,\omega t_0(\cos\theta_1 - \cos\theta_2)] ,$$
$$u_k = \sqrt{[i\,\omega t_0(1+K) + (R_0)^{-1}]^2 + (\omega t_0\sin\theta_k)^2} , \quad (10)$$
$$k = 1, 2.$$

This is the basic equation used for determination of parameters of the model from the data of the experiment and for elimination of the distorting influence of the path on the signal $f(t)$. The parameters $\xi(\omega)$ are determined in the process of detection and ranging of a source with a multiposition SIMS from the signals recorded by different spaceships. They can be calculated directly as the ratio of the spectra $F_1(\omega)$ and $F_2(\omega)$. However, to find $\xi(\omega)$, it is more convenient to apply the processor based on the fast transversal filter (FTF), which gives more accurate and reliable results. When the signals $f_i(t)$ and $f_j(t)$ come to the FTF as input and reference signals, the FTF transfer function, as FTF operates in the steady-state mode, is well approximated by the ratio $H_i(\omega)/H_j(\omega)$. The operating principle and characteristics of such a processor are described in Refs. 15 and 16.

Let us consider the case of small = = $(\omega\, t_0^2 \sin\theta)^2$, assuming = $\lesssim 1$. First, it is convenient to approximate Eq. (10) for small = as $\xi(\omega)/\xi(0) = 1 + l_1(\omega) - il_2(\omega)$. Here

$$M_1(\omega) = [(\omega t_0)^2/2]\, w_1(\omega)\, (\sin^2\theta_2 - \sin^2\theta_1)\,; \quad (11)$$

$$M_2(\omega) = [(\omega\, t_0)^2/2]\, w_2(\omega)\, (\sin^2\theta_2 - \sin^2\theta_1) + \omega\, t_0\, (\cos\theta_2 - \cos\theta_1)\,; \quad (12)$$

$$w_1(\omega) = \{[\omega\, t_0\, (1 + K)]^2\, [(R_0)^{-1} - 1] + [1 + (R_0)^{-1}]\, (R_0)^{-2}\}\, \{[\omega t_0 (1 + K)]^2 + (R_0)^{-2}]\}^{-1}, \quad (13)$$

$$w_2(\omega) = \omega t_0 (1 + K)\{(R_0)^{-1} + (R_0)^{-2} + [(\omega t_0)^2 (1 + K)]\}. \quad (14)$$

The values of $l_1(\omega)$ and $l_2(\omega)$, as well as of θ_1 and θ_2, are thought known from the experiment (experimental data on detection and ranging of a source and determination of the angles θ_1 and θ_2 can be processed, for example, by the technique from Ref. 17). At $(\omega t_0)^2 \ll 1$ we can calculate t_0 from Eq. (11) and R_0 from Eq. (12). Then it is easy to find K with $(\omega t)^2 \approx 1/2$. The estimates obtained from Eqs. (11) and (12) are then checked using Eq. (10).

With $K = 0.73(H/h)$ and $t_0 = h/c$ known, we calculate H and h. Upon determination of H and h and refinement (if necessary) of R_0, the model of the path given by Eqs. (7) and (8) becomes fully determined, and the signal $f(t)$ is reconstructed.

5. Reconstruction of the initial shape $f(t)$ of the signal emitted by a source using the above-described scheme of deconvolution of equations (2) with the output data of the system of detection and ranging eliminates the perturbation effect of the propagation path on the SIMS functioning conditions and enhances the stability of system operation under natural conditions at varying meteorological situation.

The data obtained in deconvolution of Eqs. (2) can be used to suppress noise components of the recorded signals and to correct the values of the angles θ_i, $i = 1, \ldots, N$, determined in the process of detection and ranging.

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