DETERMINATION OF THE AXIALLY SYMMETRIC ELONGATED PARTICLES ORIENTATION FROM DATA OF POLARIZATION SOUNDING

D.N. Romashov and R.F. Rakhimov

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk
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This paper deals with the analysis of influence of a preferred orientation of axially symmetric elongated particles on the backscattering phase matrix formation. Based on the analysis of relations between the elements of backscattering phase matrix a possibility is revealed of determining the angle and the degree of preferred orientation of axially symmetric particles.

1. BACKSCATTERING PHASE MATRIX (BPM) OF AN ENSEMBLE OF AXIALLY SYMMETRIC ELONGATED PARTICLES (ASEP)

The prospects of the polarization technique of sounding the atmosphere are mainly connected with the possibility of the optically detecting processes of appearance and change of the orientation of aerosol particles aimed at determining the intensity and direction of air flows at different altitudes.

The question on the influence of preferred orientation of cylindrical particles on the polarization state of lidar returns (Stokes parameters) was considered in Refs. 1 and 2 based on model estimates. This paper deals with further studies of this subject from the standpoint of particles orientation influence on the BPM elements.

Let us consider an ensemble of horizontally oriented ASEP (needles, spheroids, ellipsoids, hexagonal columns), i.e., the particles that have a plane of mirror symmetry perpendicular to the particles symmetry axis. To specify the discussion below we shall analyze only the case with circular cylinders, though the final results are valid in the majority of cases with other particles mentioned above.

Let us consider the coordinate system (x, y, z), (r, θ, φ) where the z axis coincides with the direction of radiation incidence and the polarization state of incident radiation is set in the plane (x, z), or $\varphi = 0$. The particle orientation, i.e., the orientation of symmetry axis, is given by the polar angles (α, β) (Fig. 1). The axes of particles elongation are oriented in the plane perpendicular to the direction of radiation incidence, i.e., $\beta = \pi/2$.

Let the orientation of the cylinder with the radius r and length l be determined by the angle α. Let the plane $\varphi = \alpha$ be the reference plane. It is known[3,4] that the BPM $\mathbf{F}(\alpha)$ of ASEP with respect to this plane and for $\theta = \pi$ has the form

$$\mathbf{F}(r, l, \alpha) = \begin{pmatrix} A & B & 0 & 0 \\ B & A & 0 & 0 \\ 0 & 0 & C & -D \\ 0 & 0 & D & C \end{pmatrix}, \quad (1)$$

where the elements A, B, C, and D are the functions of r and l and do not depend on α.

The backscattering phase matrix for the same particle $\mathbf{Z}(\alpha, r, l)$ but with respect to the reference plane $\varphi = 0$ can be derived from matrix (1) by means of the transformation[5]

$$\mathbf{Z}(r, l, \alpha) = \mathbf{L}(-\alpha)\, \mathbf{F}(r, l, \alpha)\, \mathbf{L}(-\alpha) . \quad (2)$$

Analogous transformation (with small correction for the angle sign) can be done for the forward scattering phase matrix $(\theta = 0)$

$$\mathbf{S}(r, l, \alpha) = \mathbf{L}(\alpha)\, \mathbf{Y}(r, l, \alpha)\, \mathbf{L}(-\alpha) , \quad (3)$$

where $\mathbf{L}(-\alpha)$ is the matrix of transformation of Stokes parameters for the case of the reference plane rotation by an angle α clockwise, if one looks along the wave propagation direction

$$\mathbf{L}(-\alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\alpha & -\sin 2\alpha & 0 \\ 0 & \sin 2\alpha & \cos 2\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

and the matrix $\mathbf{Y}(r, l, \alpha)$ is analogous to $\mathbf{F}(r, l, \alpha)$ but at $\theta = 0$

$$\mathbf{Y}(r, l, \alpha) = \begin{pmatrix} G & E & 0 & 0 \\ E & G & 0 & 0 \\ 0 & 0 & H & -T \\ 0 & 0 & T & H \end{pmatrix}.$$

Using expressions (1) and (2) we obtain

$$\mathbf{Z}(r, l, \alpha) = \begin{pmatrix} A & B\cos 2\alpha & -B\sin 2\alpha & 0 \\ B\cos 2\alpha & M_1 & -N\sin 4\alpha & D\sin 2\alpha \\ B\sin 2\alpha & N\sin 4\alpha & M_2 & -D\cos 2\alpha \\ 0 & D\sin 2\alpha & D\cos 2\alpha & C \end{pmatrix}. \quad (4)$$

where $M_1 = A\cos^2 2\alpha - C\sin^2 2\alpha$, $M_2 = C\cos^2 2\alpha - A\sin^2 2\alpha$, and $N = \dfrac{(A + C)}{2}$.

Let us now consider a monodisperse ensemble of cylindrical particles, the axes of which have a preferred orientation α_0. Let $f(\alpha, \alpha_0, \kappa)$ be a function of the distribution density of orientations of particles axes over the angle α. Total backscattering phase matrix of an ensemble of spatially oriented elongated particles with respect to the reference plane $\varphi = 0$ can be obtained by integrating over all planes α with the weighting function $f(\alpha, \alpha_0, \kappa)$

$$\mathbf{P}(r, l, \alpha_0) = \int_0^{\pi} \mathbf{Z}(r, l, \alpha)\, f(\alpha, \alpha_0, \kappa)\, d\alpha . \quad (5)$$

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Let us determine the distribution of particle axes orientations over the angle α via the Mises distribution function[6]

$$f(\alpha,\,\alpha_0,\,\kappa) = \exp[\kappa\cos 2(\alpha-\alpha_0)]/\,\pi\, I_0(\kappa) \tag{6}$$

where $J_0(\kappa)$ is the modified zero–order Bessel function of the first kind. Mises distribution is a single peaked function symmetrical with respect to the point $\alpha_c = \alpha_0 \pmod{\pi}$. The greater is k, the narrower is the distribution around the mode. The distribution density function has two inflection points

$$\alpha_b = \alpha_0 \pm 1/2\ \arccos\left(1/2\kappa + \sqrt{1 + 1/4\ \kappa^2}\right)$$

in the interval $(\alpha_0 - \pi/2,\ \alpha_0 + \pi/2)$.

Note that the density function of normal distribution $N(\mu,\,\sigma)$ has the inflection points at $\alpha = \mu \pm \sigma$. Thus, the value $\alpha_k = \alpha_b - \alpha_0$ is a measure of particle orientations spread around the direction α_0.

After integration of Eq. (5) and using Eqs. (4) and (6) we obtain for the elements of BPM $P(r,\,l,\,\alpha_0)$ of an ensemble of polyoriented particles

$$\begin{pmatrix} A & i_1B\cos 2\alpha_0 & -i_1B\sin 2\alpha_0 & 0 \\ i_1B\cos 2\alpha_0 & U + i_2N\cos 4\alpha_0 & -i_2N\sin 4\alpha_0 & i_1D\sin 2\alpha_0 \\ i_1B\sin 2\alpha_0 & i_2N\sin 4\alpha_0 & -U + i_2N\cos 4\alpha_0 & -i_1D\cos 2\alpha_0 \\ 0 & i_1D\sin 2\alpha_0 & i_1D\cos 2\alpha_0 & C \end{pmatrix}, \tag{7}$$

where $U = \dfrac{(A-C)}{2}$, $i_1(\kappa) = \dfrac{I_1(\kappa)}{I_0(\kappa)}$, $i_1(\kappa) = \dfrac{I_2(\kappa)}{I_0(\kappa)}$, I_0, I_1, I_2 are the modified zero–, first–, and second–order Bessel functions of the first kind, respectively.

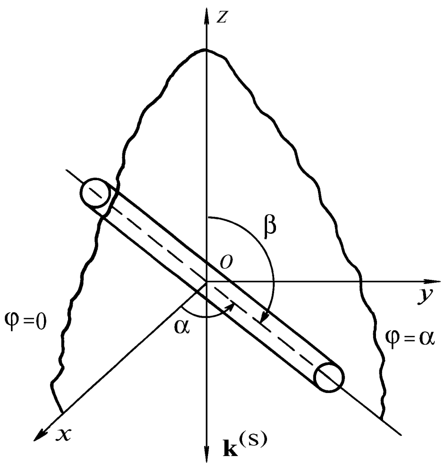

FIG. 1. Geometry of light scattering by an arbitrarily oriented circular cylinder.

All the above considerations have been undertaken for the case of an ensemble of polyoriented elongated cylindrical particles with fixed size l and r. In the case of elongated particles we have $l \gg r$, and therefore it is physically correct to suppose that the particles of such a shape but different size take, under the action of horizontal air flows, approximately the same orientation, i.e., the particle size distribution is independent of the particle orientation distribution function. In this case the elements of BPM of a polydisperse ensemble of particles are usually estimated by averaging over the size spectrum with the weighting function $g(r,\,l)$

$$\overline{\mathbf{P}}(\alpha_0) = \int_{l_1}^{l_2}\int_{r_1}^{r_2} \mathbf{P}(r,\,l,\,\alpha_0)\ g(r,\,l)\ \mathrm{d}l\ \mathrm{d}r\ . \tag{8}$$

Thus, the dependence on α_0 is the same as that for a monodisperse ensemble

$$\overline{\mathbf{P}}(\alpha_0) = \begin{pmatrix} \overline{A} & i_1\overline{B}\cos 2\alpha_0 & -i_1\overline{B}\sin 2\alpha_0 & 0 \\ i_1\overline{B}\cos 2\alpha_0 & \overline{U} + V & -i_2\overline{N}\sin 4\alpha_0 & i_1\overline{D}\sin 2\alpha_0 \\ i_1\overline{B}\sin 2\alpha_0 & i_2\overline{N}\sin 4\alpha_0 & -\overline{U} + V & -i_1\overline{D}\cos 2\alpha_0 \\ 0 & i_1\overline{D}\sin 2\alpha_0 & i_1\overline{D}\cos 2\alpha_0 & \overline{C} \end{pmatrix}. \tag{9}$$

where $\overline{N} = (\overline{A} + \overline{C})/2$, $\overline{U} = (\overline{A} - \overline{C})/2$, and $V = i_2\overline{N}\cos 4\alpha_0$.

The scattering phase matrix $\overline{S}(\alpha_0)$ for the case of forward scattering has the form

$$\begin{pmatrix} \overline{G} & i_1\overline{E}\cos 2\alpha_0 & -i_1\overline{E}\sin 2\alpha_0 & 0 \\ i_1\overline{E}\cos 2\alpha_0 & K - i_2W\cos 4\alpha_0 & i_2W\sin 4\alpha_0 & -i_1\overline{T}\sin 2\alpha_0 \\ -i_1\overline{E}\sin 2\alpha_0 & i_2W\sin 4\alpha_0 & K + i_2W\cos 4\alpha_0 & -i_1\overline{T}\cos 2\alpha_0 \\ 0 & i_1\overline{T}\sin 2\alpha_0 & i_1\overline{T}\cos 2\alpha_0 & \overline{H} \end{pmatrix}. \tag{10}$$

where $K = (\overline{H} + \overline{G})/2$, $W = (\overline{H} - \overline{G})/2$.

In our further discussion we shall omit the averaging bar above the matrices and their elements while meaning, at the same time, that we deal with polydisperse ensembles. Note also that

$$\lim_{\kappa\to 0} i_{1,2}(\kappa) = 0, \tag{11}$$

as a result, Mises distribution becomes uniform, and the BPM takes the diagonal form

$$P = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & U & 0 & 0 \\ 0 & 0 & U & 0 \\ 0 & 0 & 0 & C \end{pmatrix}, \tag{12}$$

what well agrees with the known conclusion drawn in Ref. 3.

2. THE TECHNIQUE FOR DETERMINATION OF THE ANGLE OF PREFERRED ORIENTATION AND OF THE DEGREE OF ASEP ORIENTATION

From matrix (9) we can derive the following calculational relationships between the elements of BPM and distribution parameters α_0 and κ:

$$P_{11} - P_{22} = P_{44} - P_{33}\ ; \tag{13}$$

$$\cot 2\alpha_0 = -\frac{P_{12}}{P_{13}}\,,\quad \cot 2\alpha_0 = -\frac{P_{34}}{P_{24}}\ ; \tag{14}$$

$$i_2(\kappa) = \frac{(P_{22} + P_{33})}{(P_{11} + P_{44})\cos 4\alpha_0}; \quad \alpha_0 \neq \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}; \quad (15)$$

$$i_2(\kappa) = \frac{2P_{32}}{(P_{11} + P_{44})\sin 4\alpha_0}; \quad \alpha_0 \neq 0, \frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}. \quad (16)$$

Relation (13) can serve as a criterion of the BPM measurement correctness. The validity of this criterion can be shown not only for elongated particles but also for particles of an arbitrary shape.

Relationships (14) make it possible to estimate the angle of preferred orientation of particles. However, it often occurs in measurement practice so that the values P_{12} and P_{13} or P_{34} and P_{24} are close to zero. Therefore, it is advisable to use in such cases, for determination of α_0, the elements that essentially differ from zero. Taking into account that α_0 varies in the interval $(0 - \pi)$ from (14) we obtain two values of α_0 that differ by $\pi/2$

$$\alpha_0^1 = \frac{1}{2}\,\mathrm{arccot}\left(-\frac{P_{12,\,34}}{P_{13,\,24}}\right);$$

$$\alpha_0^2 = \frac{1}{2}\,\mathrm{arccot}\left(-\frac{P_{12,\,34}}{P_{12,\,34}}\right) + \frac{\pi}{2}. \quad (17)$$

For the sake of simplicity of determining α_0 one should take into account that for the infinite cylinders with the axes laying in the scattering plane the elements of BPM (1) B and D are negative.

If $P_{12} > 0$ $(P_{43} > 0)$ then $\pi/4 < \alpha_0 < 3\pi/4$. In the opposite case of $P_{12} < 0$ $(P_{43} < 0)$ we have $0 < \alpha_0 < \pi/4$ or $3\pi/4 < \alpha_0 < \pi$.

Relationships (15) and (16) complement each other and are useful for estimating the degree of particles orientation. To do this, it is sufficient to make a table of the function $i_2(\kappa)$ in the interval $(0 - 10)$ (since $i_2(0) = 0$ and $i_2(10) \approx 1$) in order to estimate the parameters κ and α_κ of the Mises distribution by the calculated function $i_2(\kappa)$. These parameters determine the degree of particle axes spread around α_0. Since the sum $P_{11} + P_{44}$ does not depend on the orientation type, it follows from (15) and (16) that the orientation degree is directly proportional either to $P_{22} + P_{23}$ or to P_{32}. And, as it is seen from the elements of matrix (4), relationships (15) and (16) are reduced to the equality $i_2(\kappa) = 1$ in the case of strictly oriented particles.

The statement by the authors of Ref. 7 that one can use the value $\sqrt{1 - S_4^2}$ for estimating the degree of orientation seems to be incorrect. Here S_4 is the fourth normalized Stokes parameter of scattered radiation in the case when medium is irradiated by circularly polarized radiation and measurements are being done at the scattering angles $\theta \approx 10$ and 170°. In fact, the value S_4 for circularly polarized incident radiation characterizes $(P_{14} \mp P_{44})$, where P_{14} is a zero valued element of the scattering phase matrix, and, according to the conclusions drawn by the authors and as it follows from our results [see expressions (9) and (10)] P_{44} does not depend on the particles orientation. For this reason the change of this value recorded by the authors in their measurements is evidently connected with the change of the degree of particles nonsphericity under the action of orienting magnetic field but not of the orientation itself.

3. PROPERTIES OF THE ASEP BPM

Measurements of the backscattering phase matrix of essentially nonspherical aerosol particles are known primarily from laboratory investigations. Nevertheless, it is useful to compare our results with the data for elongated circular cylinders of γ–Fe_2O_3 presented in Ref. 7.

The following theoretical conclusions that can be drawn from our results are in a good agreement with the data from Ref. 7:

(a) $P_{14} = 0$ for all α_0,

(b) P_{44} does not depend on α_0, and P_{33} depends on α_0,

(c) for $\alpha_0 = 0$ the BPM has the form

$$\mathbf{P}(0) = \begin{pmatrix} A & i_1 B & 0 & 0 \\ i_1 B & U + i_2 N & 0 & 0 \\ 0 & 0 & -U + i_2 N & -i_1 D \\ 0 & 0 & i_1 D & C \end{pmatrix}. \quad (18)$$

The BPM for $\alpha_0 = \pi/2$ has analogous structure. The difference is that the elements P_{12} (P_{21}) and P_{34} (P_{43}) change sign. At the beginning of their article the authors of Ref. 7 arrived at the same conclusion, but when analyzing the behavior of P_{12} they concluded that this element of BPM was negative for all α_0.

Among other most interesting results following from the view of BPM one can point out the following:

(a) For $\alpha_0 = \pi/4$ the BPM has the form

$$P\left(\frac{\pi}{4}\right) = \begin{pmatrix} A & 0 & -i_1 B & 0 \\ 0 & U - i_2 N & 0 & i_1 D \\ i_1 B & 0 & -U - i_2 N & 0 \\ 0 & i_1 D & 0 & C \end{pmatrix}. \quad (19)$$

The BPM for $\alpha_0 = 3\pi/4$ has the similar structure. Difference is that the elements P_{13} (P_{31}) and P_{24} (P_{42}) have opposite signs.

(b) For $\alpha_0 = \pi/8$ the BPM has the form

$$P\left(\frac{\pi}{8}\right) = \begin{pmatrix} A & \frac{\sqrt{2}}{2} i_1 B & -\frac{\sqrt{2}}{2} i_1 B & 0 \\ \frac{\sqrt{2}}{2} i_1 B & U & -i_2 N & \frac{\sqrt{2}}{2} i_1 D \\ \frac{\sqrt{2}}{2} i_1 B & i_2 N & -U & -\frac{\sqrt{2}}{2} i_1 D \\ 0 & \frac{\sqrt{2}}{2} i_1 D & \frac{\sqrt{2}}{2} i_1 D & C \end{pmatrix}. \quad (20)$$

For $\alpha_0 = 5\pi/8$ the BPM is similar, only the elements $P_{13}(P_{31})$, $P_{24}(P_{42})$, $P_{12}(P_{21})$, and P_{34} (P_{43}) have opposite signs.

(c) For $\alpha_0 = 3\pi/8$ the BPM has the form

$$P\left(\frac{3\pi}{8}\right) = \begin{pmatrix} A & -\frac{\sqrt{2}}{2} i_1 B & -\frac{\sqrt{2}}{2} i_1 B & 0 \\ -\frac{\sqrt{2}}{2} i_1 B & U & i_2 N & \frac{\sqrt{2}}{2} i_1 D \\ \frac{\sqrt{2}}{2} i_1 B & -i_2 N & -U & \frac{\sqrt{2}}{2} i_1 D \\ 0 & \frac{\sqrt{2}}{2} i_1 D & -\frac{\sqrt{2}}{2} i_1 D & C \end{pmatrix}$$

.(21)

The structure of the BPM at $\alpha_0 = 7\pi/8$ is similar to this one except for signs of the elements P_{13} (P_{31}), P_{24} (P_{42}), P_{12} (P_{21}), and P_{34} (P_{43}), which are opposite.

Two curves of P_{12} and P_{13} for $\alpha_0 = 70$ were presented in Ref. 7. Their behaviors at the scattering angles θ close to 170 are in a good agreement with the theoretical estimates in Eq. (17).

Measurement results presented in Ref. 7 confirm (accurate to a minus sign) relationship (13) for diagonal elements of the scattering phase matrix. This relationship is also obtained in theoretical calculations for spheroids[5] and hexagonal crystals[8] as well as it takes place in experiments with ice fogs.[8,9]

Thus, the analysis made shows that although the orientation anysotropy of nonspherical particles can result in essential variations of the aerosol scattering parameters, it is possible to detect reliably the presence and degree of the preferred orientation of ASEP in a specially arranged field experiment, using the properties of BPM (relationships between the BPM elements).

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