

INTERPRETATION OF GROUND-BASED RADIOMETRIC MEASUREMENTS IN THE 0.8–1.35 CM REGION INCLUDING A PARAMETRIZATION OF SYSTEMATIC ERRORS

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A simple parametrization consisting of linear functions of the brightness temperature corresponding to the "average" state of the atmosphere is proposed for the purpose of representing certain components of the systematic error which may occur regardless of radiometer design during spectral-angular measurements of the brightness temperature from the downwelling radiation in the wavelength region 0.81–1.35 cm. The accuracy of the proposed approach in eliminating the influence of such systematic errors is investigated using the results of numerical experiments and of processing SHF radiometric measurements. In particular, it is shown that the suggested method makes it possible to solve an essentially nonlinear inverse problem in the remote sensing of humidity without employing any special iteration algorithms.

INTRODUCTION

Radiometric method have been actively used for a fairly long time to determine various parameters of the state of both the cloudy and the cloudless atmosphere. In particular, measurements of the downwelling microwave thermal radiation in the 0.8–1.35 cm band wind extensive application in the retrieval of humidity profiles, the integral water vapor content, and the cloud water content.^{1–6}

To interpret the ground-based radiometric measurement data in this spectral range, the equation of microwave radiation transfer is solved in the absence of scattering (Cloudy atmosphere without precipitation). The solution may be written in the following form:¹

$$T_{br} \left[\lambda, \theta, T, q, \omega, T_f \right] = \frac{1}{\cos \theta} \int_0^{\infty} \alpha_{\lambda}(z, T, q, \omega) \times \\ \times T(z) \exp \left[- \frac{1}{\cos \theta} \int_0^z \alpha_{\lambda}(z', T, q, \omega) dz' \right] dz + \\ + T_f(\lambda) \exp \left[- \frac{1}{\cos \theta} \int_0^{\infty} \alpha_{\lambda}(z, T, q, \omega) dz \right], \quad (1)$$

where z is height; λ is wavelength; θ is the zenith angle; T is the thermodynamic temperature; q is the humidity; ω is cloud water content; T_{br} is the brightness temperature of the downwelling radiation at the underlying surface level; α_{λ} is the absorption coefficient; and $T_f(A)$ is the brightness temperature of

the background radiation, the dependence of which on the wavelength in the examined spectral range may be neglected.⁹

By virtue of the nonlinearity of Eq. (1) with respect to the sought-after atmospheric meteorological parameters, we first construct a linearized analog of the solution of the inverse problem. Specific expressions for such a linearized equation of transfer may be found, e.g., in Refs. 1 and 4. If measurement data on the downwelling microwave thermal radiation are available for some set of wavelengths and zenith angles, the retrieval of the vertical profile of any meteorological element (the so-called independent problem) or the joint retrieval of a combination of profiles (the complex problem)² reduces to solving the following linear finite-dimensional analog of Eq. (1):

$$\vec{\delta T}_{br} = A_x \vec{\delta x} + \vec{\epsilon}, \quad (2)$$

where $\vec{\delta T}_{br}$ is a column vector of dimension $L \cdot v$ whose components are the deviations of the measured values of the brightness temperature from its respective averages at wavelength L and zenith angles v ; $\vec{\delta x}$ is a column vector whose components are the deviation of the sought-after parameter from its respective averages at N levels in the atmosphere (when performing a complex retrieval $\vec{\delta x}$ is the joint vector of variations of all the sought-after meteorological elements)²; A_x is a matrix that approximates the corresponding linear integral operator; ϵ is the vector of measurement errors, which may be represented as sum of the systematic and random components ($\vec{\epsilon} = \vec{\epsilon}_s + \vec{\epsilon}_r$).

The method of solving ill-posed inverse problems which has been developed to date enable one to

construct a solution only if an exclusively random measurement error is presented on the right-hand side of Eq. 2 (Ref. 8). An account of the systematic component is usually complicated by its dependence on the unknown state of the atmosphere.

This systematic component may be accounted for in the solution algorithm for the inverse problem in two alternative ways: a) construct an estimate $\vec{\delta x}$ with operator A_x , first neglecting the systematic component, and then estimating its influence on the accuracy of the solution; b) try to estimate the systematic component together with the sought-after vector $\vec{\delta x}$ in one and the same experiment. To realize either approach, a priori data are needed on the systematic component of the measurement error.

The object of the present paper is to study the possibilities of the second approach basing it on the parametrization of the components of the systematic measurement error. It is illustrated by the solution of the inverse problem of retrieving the vertical humidity profile for a cloudless atmosphere. In that case Eq. (2) assumes the form

$$\vec{\delta T}_{br} = A \vec{\delta q} + \vec{\epsilon} \tag{3}$$

BASIC RELATIONSHIPS

A comprehensive analysis of the complete set of systematic components of the measurement error is possible only for some particular radiometric apparatus under known conditions of its use. We therefore distinguish only those components of the systematic error which may to some extent appear when measuring the microwave brightness temperature irrespective of the radiometer design. These are, first of all, the errors that are due to inaccuracies in the positioning of the zenith angle ϵ_θ ; secondly, the errors that are due to the presence of the cosmic background ϵ_f ; thirdly, calibration errors ϵ_c ; and, finally, linearization errors ϵ_l , which we shall formally interpret as a component of the radiometric measurement error. Since their dependence on the unknown state of the atmosphere is quite complicated, they are best parametrized in the measurement space (the space of the measurements of the brightness temperatures).

With this purpose in mind, let us examine the characteristic angular trend of the systematic components of the measurement error and the "informative" signal — $f(\theta)$, i.e., the brightness temperature variations caused only by variations of the humidity vertical profile, specified on the basis of the following relationships:

$$\epsilon_f(\theta) = T_{br}[\lambda, \theta, \bar{q}, T_{f1}] - T_{br}[\lambda, \theta, \bar{q}, T_{f2}]; \tag{4}$$

$$\epsilon_\theta(\theta) = T_{br}[\lambda, \theta + \Delta\theta, \bar{q}, T_{f1}] - T_{br}[\lambda, \theta, \bar{q}, T_{f1}]; \tag{5}$$

$$f(\theta) = T_{br}[\lambda, \theta, \bar{q} + \delta q, T_{f1}] - T_{br}[\lambda, \theta, \bar{q}, T_{f1}]; \tag{6}$$

$$\epsilon_c(\theta) = \gamma_c \left[T_c - T_{br}[\lambda, \theta, \bar{q}, T_{f1}] \right]; \tag{7}$$

$$\epsilon_l(\theta) = \delta T_{br}[\lambda, \theta, \delta q, T_{f1}] - f(\theta), \tag{8}$$

where $T_{br}[\dots]$ is evaluated numerically by integrating Eq. (1) for the appropriate values of the arguments values, and $\delta T_{br}[\dots]$, by numerically integrating its linearized analog; T_c is the temperature of the surface layer; γ_c depends on the zenith brightness temperature; \bar{q} is the mean humidity profile; δq is its variation; and $\Delta\theta$ is the zenith angle variation.

Of the above relationships (4)–(8) only the equation for the calibration error requires separate discussion. Its derivation we based on the fact that field calibrations during radiometric measurements are very conveniently made by taking just two readings, namely, of the horizon and zenith brightness temperatures.^{1,6} Note that the zenith brightness temperature may be specified only very approximately. If the measured horizon temperature is assumed to be known and equal to that of the surface layer, the calibration error associated with the uncertainty in the zenith brightness temperature may be represented by Eq. (7). That formula is a particular case of an equation which describes the total error of the radiometric measurements when the absolute calibration is made against two references. A detailed derivation and analysis of that relationship is given in Ref. 5. Note that the "horizon" measurements are, in their turn, complicated by the fact that the side lobes of the directional diagram hit the underlying surface, which may lead to considerable errors. In principle, however, the suggested approach may account for the errors both in one and in two calibration points. This would require introducing an additional parameter and modifying Eq. (7).

Results of calculating the components of the systematic errors and the "informative" signal are given in Figs. 1a and b (linearization errors for $\lambda = 0.8$ cm do not exceed a few tenths of a degree and are not shown in Fig. 1a). Analysis of the data in Fig. 1, demonstrates, first, that within the angle range $0 - 86.5^\circ$ to within ~ 1 K, the components of the systematic measurement error may be represented by linear functions of the mean brightness temperature of the atmosphere, and, second, that the angular trends of the informative signal and of the components of the systematic error in that angle range differ substantially from each other. The first circumstance makes it possible to parametrize the components of the systematic error in the following way:

$$\epsilon_f(\theta) = \gamma_f \left[T_c - \bar{T}_{br}(\theta) \right];$$

$$\epsilon_l(\theta) = \beta_l \bar{T}_{br}(\theta), \tag{9}$$

where γ_f , β_0 and β_1 are wavelength-dependent parameters, and $\bar{T}_{br}(\theta) = T_{br}[\lambda, \theta, \bar{q}, T_{f1}]$. The second circumstance points to the possibility of separating the

"informative" signal from the systematic error in the interpretation of the measurements of the atmospheric downwelling microwave thermal radiation.

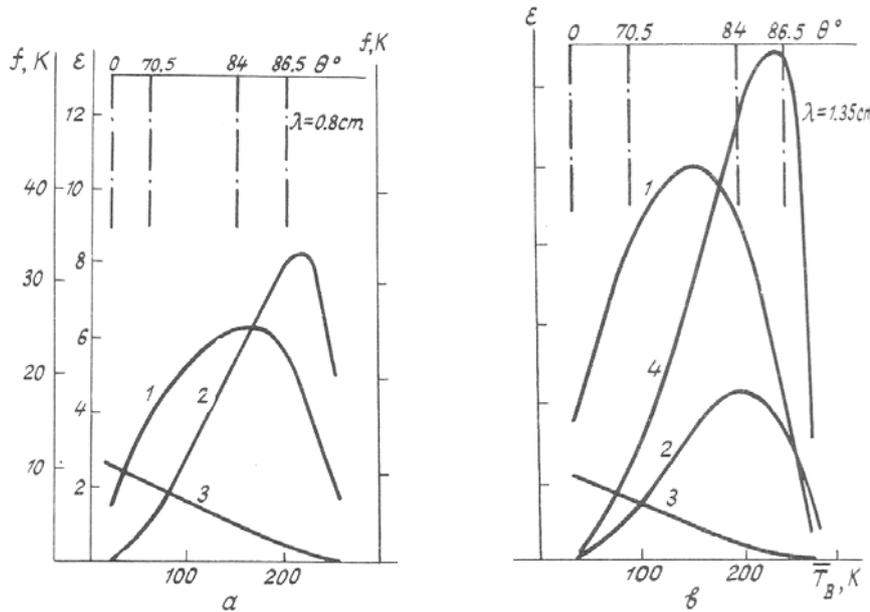


FIG. 1. Angular dependence of the "informative" signal (f) and the systematic components of the error of the near-surface radiometric measurements (ϵ_c): 1) $f(\theta)$, 2) $\epsilon_c(\theta)$, 3) $\epsilon_f(\theta)$ ($T_{f1} = 2.7$ K, $T_{f2} = 0$ K, $\Delta\theta = 0.3^\circ$, and the variations δq correspond to 40% of the mean profile).

Note that the physical reason for using the fairly simple linear parametrization (9) consists in the absence of "saturation" effects for the brightness temperature up to zenith angles very close to $\pi/2$ a consequence of the rather high atmospheric transparency in the examined spectral range. Summing up all the components of the systematic measurement error, we obtain the following equation for $\epsilon_s(\theta)$:

$$\epsilon_s(\theta) = \beta \bar{T}_{br}(\theta) + \gamma(T_c - \bar{T}_{br}(\theta)), \tag{10}$$

where β and γ depend only on λ , and not on θ ; $\beta = \beta_0 + \beta_1$ and, $\gamma = \gamma_f + \gamma_c$.

In view of Eq. (10), Eq. (3) may be written in the form

$$\vec{f} = A_q \delta \vec{q} + A_p \vec{P} + \vec{\epsilon}_r, \tag{11}$$

where P is a vector, having as its components the parameters β and γ for the wavelengths of 0.8 and 1.36 cm: $P^+ = (\beta_{0.8}, \beta_{1.35}, \gamma_{0.8}, \gamma_{1.35})$, and the matrix A_p is composed of the elements $\bar{T}_{br}(\lambda_i, \theta_j)$ and $T_c - \bar{T}_{br}(\lambda_i, \theta_j)$, $i = 1, 2; j = 1, \dots, v$.

One can see from Eq. (11) that the use of the above systematic error parametrization requires the estimation of four additional parameters together with the sought-after vector $\delta \vec{q}$. We therefore assume the vector $\delta \hat{\vec{q}}$ and $\hat{\vec{p}}$ to be a solution of

Eq. (11), i.e., they minimize the following functional, written in the energy norm:

$$\Phi = \vec{f} - A_q \delta \vec{q} - A_p \vec{P} \quad \Sigma^{-1} + \delta \vec{q} \quad D^{-1} + \delta \vec{q}_s - S \delta \vec{q} \quad \Sigma_s^{-1} + \vec{P} \quad D_p^{-1} \tag{12}$$

Here Σ is the covariance matrix of the random component of the errors in the measurement of the brightness temperature; D is the covariance matrix of the vector $\delta \vec{q}$; $\delta \vec{q}_s$ is the vector of "direct" measurement of the humidity; s is the operator of the "direct" measurements; Σ_s is the matrix of the errors of the "direct" measurements; and $D_p^{-1} = rI$, where I is the 4×4 unit matrix and $r = 10^{-4} \div 10^{-6}$. Below, as such "direct" measurements we use data on the surface-layer humidity, obtained, for example, using a "Volna-1M" hygrometer (absolute error of the relative humidity measurement $\pm 1.5\%$). In that case the vector $\delta \vec{q}_s$ is one-dimensional, the matrix s consists of one row of zero elements, except for the first, which is unity, and the matrix Σ_s contains one element, whose magnitude characterizes the variance of the direct measurements.

Solving the system of Euler equations for the functional (12), we obtain the following equation for the estimate vector $\delta \hat{\vec{q}}$:

$$\hat{\delta \bar{q}} = M A_q^+ \Sigma_e^{-1} \vec{f} + M S^+ \Sigma_s^{-1} \delta \bar{q}_s, \tag{13}$$

where

$$M = (A_q^+ \Sigma_e^{-1} A_q + S^+ \Sigma_s^{-1} S + D^{-1})^{-1}; \Sigma_e = \Sigma + k A_p D A_p^+$$

and the parameter k takes the value 0 and 1. For $k = 0$, taking into account the results of the direct humidity measurements at the surface, estimate (13) reduces to the standard estimate, obtained using the statistical regularization method; for $k = 1$, it gives the estimate $\hat{\delta \bar{q}}$ if the vector of additional parameters p is available.

THEORETICAL ESTIMATES OF ACCURACY AND THE RESULTS OF NUMERICAL EXPERIMENTS

One can see from Eq. (13) that the evaluation of the additional parameters which characterize the systematic measurement error, in this case is adequate to retrieve the vertical humidity profile with an additional effective error determined by the covariance matrix $A_p D A_p^+$. Therefore we shall first consider theoretical estimates of the accuracy of retrieval both with and without the systematic error, obtained by calculating the matrix M ($k = 1, k = 0$, respectively). The diagonal elements of $M(\sigma^2)$ characterize the retrieval variances for the sought-after humidity profile. It was assumed in the calculations that the covariance matrix D corresponds to the model statistics³ while the random measurement errors remain uncorrelated, with variances $\sigma_{br}^2 = 1 K^2$.

TABLE I. Relative errors in the determination of the humidity profile $\hat{\sigma} / \bar{q}$ (%) using the radiometric method (based on a calculation of the error matrix).

Z, km	0.2	0.6	1.0	1.8	2.6	3.4	4.2	5.0	6.0
Standard technique (k = 0)	12	16	17	19	22	23	24	26	30
Retrieval technique (k = 1)	14	18	21	21	23	26	27	30	31

Table I presents theoretical relative errors of the retrieved humidity profile following a microwave experiment ($\lambda_1 = 0.8$ cm, $\lambda_2 = 1.35$ cm, $\theta_{1-8} = 0.45; 60; 70.5; 75.5; 78.5; 84; 86.5^\circ$) taking into account the direct surface measurement data. Note that the errors in the retrieval of the profile $q(z)$ from the surface measurements alone, using the statistical extrapolation technique, are significantly higher and already reach ~ 30% at a height of about 1.5 km.

It should be noted that the estimated retrieval errors, obtained for the standard procedure of the

inversion of Eq. (3), are, generally speaking, underestimated, since only the random component of the radiometric measurement error is taken into account. In this sense they characterize the limiting (potential) retrieval accuracy. The error matrix, calculated for $k = 1$, serves as a more adequate characteristic of the overall accuracy since it reflects, though approximately, the effect of systematic errors.

As can be seen from Table I, in the case when $\hat{\delta \bar{q}}$ and the component of the error parametrization are jointly retrieved and the overall accuracy in the retrieval of the humidity profile differs only slightly from the potential accuracy, the discrepancies amounting to just a few percent. The closeness of the errors $\hat{\delta \bar{q}}$ for the two considered techniques confirms the above conclusion of the possibility of separating the "informative" signal from the systematic errors component.

Now let us discuss the results of numerical simulations, the purpose of which is to demonstrate the suggested methods both in the presence of the above components of systematic error and in their absence.

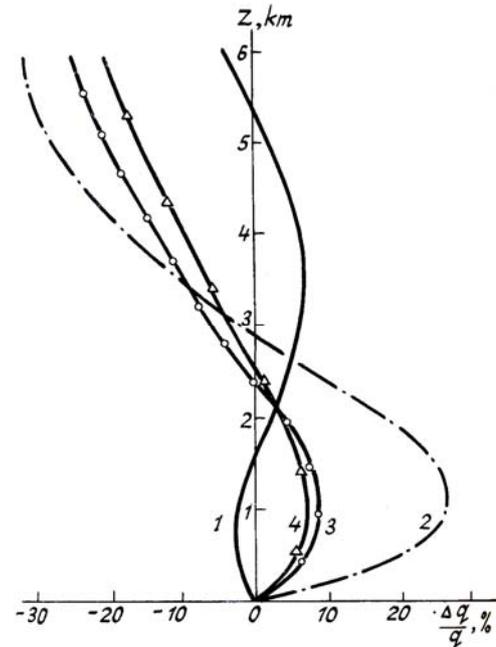


FIG. 2. Relative errors in the retrieval of the humidity profile (numerical simulation): 1, 2) standard retrieval technique in the absence (1) and presence (2) of systematic errors; 3, 4) present retrieval technique in absence (3) and presence (4) of systematic errors.

Two sets of brightness temperatures, calculated according to Eq. (1) served as initial data for these numerical simulations. The first set of "measurements" simulated the situation in which both the random and the systematic component of the measurement error are present. The systematic error values in the 0.8 and 1.35 cm bands corresponded to those given in Fig. 1, and the zenith brightness temperature calibration error was assumed to reach 3 K. The second set of

"measurements" contained only the random component of the measurement error.

Figure 2 shows the errors in the retrieval of the vertical humidity profile ($\Delta q/q \cdot 100\%$) obtained by standard processing of both sets of initial data (curves 1 and 2) and using the present method (curves 3 and 4). Note that when using the standard method the errors due to nonlinearity were neutralized using the iteration algorithm described in Ref. 7.

The analysis of the results of the numerical simulations, shown in Fig. 2, makes it possible to make the following conclusions:

1. If systematic errors are present in the brightness temperature data, the errors in the standard retrieval of the humidity profile may be several times as large as in the absence of these errors (see curves 1 and 2 in Fig. 2).

2. Employing the suggested techniques to exclude the systematic error makes it possible in case systematic errors are present in the results of the brightness temperature measurements, to reduce by a factor of almost 2–3 the errors in the retrieved water vapor profile, as compared to that obtained using the standard retrieval (cf. curves 2 and 4).

3. The absence of systematic errors in the brightness temperature measurements has practically no effect on the error level in the retrieved water vapor profile (cf. curves 3 and 4), thus demonstrating the high "selectivity" of the suggested approach with respect to systematic errors of the given type.

4. When systematic errors are present, the errors of retrieving $q(z)$, nevertheless, exceed the "potential" error limit. However, as one can see by comparing curves 1 and 4, this increase in the retrieval error does not exceed 10%. Such an increase seems to be quite natural since the same number of brightness temperature measurements is now used to estimate more parameters than before.

Generally speaking, the scheme for solving the inverse problem solution with systematic errors taken into account does not envisage the separate retrieval of all the components of the systematic error. However, out of methodological considerations, we have analyzed the accuracies of retrieving these various components. The components ε_f , ε_1 , and ε_c were retrieved with fairly high accuracy ~ 0.3 K; ~ 1 K, and 0.3 K, respectively. This attests, in particular, to the possibility of doing without the iteration procedure of kernel correction⁷ (its purpose is to eliminate linearization errors) if the present techniques are used to interpret radiometric data in the 0.8–1.35 cm range. Considerably larger retrieval errors are typical of ε_0 and may reach a few degrees. Thus when using the suggested methods one should pay special attention to the angular referencing of the measurements.

INTERPRETATION OF FIELD EXPERIMENT DATA

The suggested techniques for parametrization of the systematic radiometric measurements error components

and for their direct inclusion in the inverse problem solution algorithm were tested in the interpretation of data from a series of ground-based SHF radiometric measurements at the Voeikov Main Geophysical Observatory (MGO) at the Karadag experimental site. Measurements were taken during summer, 1986, employing an instrumentation complex that operates at the wavelengths $\lambda_1 = 0.8$ cm and $\lambda_2 = 1.35$ cm and the zenith angles $\theta = 0, 45, 60, 70.5, 75.5, 78.5, 84,$ and 86.5° .

Turning now to a discussion of the results of processing of the experimental data, it should be noted that, generally speaking, the error component may display both systematic and random features, depending on the particular mode of angular scanning. Assuming ε_0 to be random, the effective error matrix takes the form

$$\Sigma_e = \Sigma + \Sigma_r^\theta + k A \begin{matrix} D \\ P \end{matrix} A^* \quad (14)$$

Here Σ_r^θ is a diagonal matrix whose elements are calculated from a priori estimates of the errors produced by inaccurate setting of the zenith angle.

We therefore interpreted the field data following the suggested approach and assuming the error ε_0 to be both systematic and random.

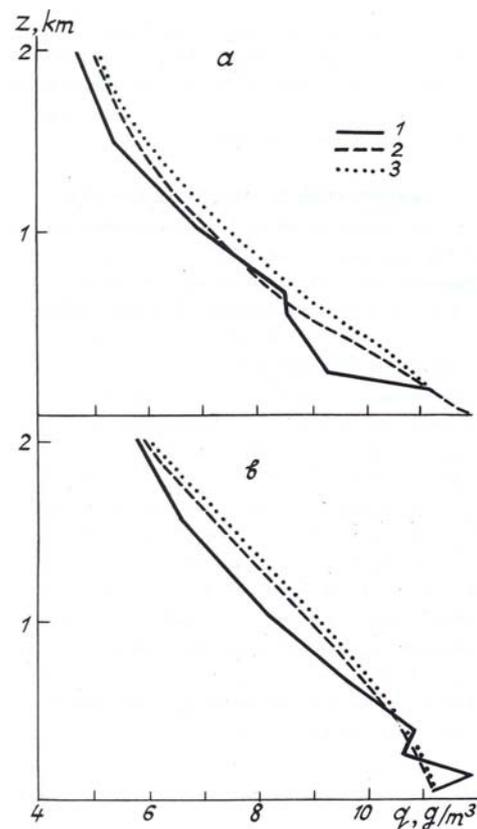


FIG. 3. Retrieval of the humidity profile (from the Voeikov MGO Karadag experimental site): a) clear sky, 07.06.86; b) overcast, 02.06.86; 1) aircraft sounding; 2) standard retrieval technique; 3) present retrieval technique.

The results of retrieving the humidity profiles (clear sky) and the profiles of the humidity and total water content (overcast) may be found in Ref. 3. These were obtained by the standard iteration procedure of kernel correction⁷ using Σ_e in the form (14) for $k = 0$. When processing the experimental results in Ref. 3 the value $T_f = 2.7$ K was explicitly taken into account in the step preceding the solution of the inverse problem.

Figure 3 illustrates the results of comparing the data in Ref. 3 and the humidity profiles retrieving using the methods suggested in the present paper. In the latter case ε_f was also excluded at a preliminary stage. When solving the inverse problem the total systematic error ε_c was estimated simultaneously with retrieval of the humidity profiles. Analysis of the results demonstrates the following:

1. If we assume a random character of the error ε_θ , then the obtained humidity profiles (curves 3, Fig. 3) coincide with the standard retrieval technique (curves 2) to within a few percent. At $\lambda = 0.8$ cm the values of $\varepsilon_r(\theta)$ do not exceed 0.5–1.0 K, while at $\lambda = 1.35$ cm they increase with θ , reaching ~ 10 K at $\theta = 86.5^\circ$.

The obtained results show that the presented technique effectively excludes the errors associated with the nonlinearity of the problem of humidity sensing. Comparing the values of $\varepsilon_r(\theta)$ at $\lambda = 0.8$ and $\lambda = 1.35$ cm we conclude that in the given experiment ε_r is mainly determined by linearization errors.

2. Discrepancies between the vertical humidity profiles, retrieved assuming the systematic character of the error ε_θ ($\Sigma_r^0 = 0$), and those measured from aircraft reach 20–30%, during the considered microwave experiment the angular scanning errors were random.

3. These results demonstrate feasibility of the suggested procedure for the case of combined retrieval of humidity and water content for weak cloud cover (the results of determining $q(z)$ given in Fig. 3b correspond to a cloud cover with liquid water content of $4.5 \cdot 10$ kg/m²).

MAIN CONCLUSION

On the basis of an analysis of the angular structure of certain components of the systematic error of

ground-based radiometric measurements in the 0.8–1.35 cm range, a parametrization for them is presented and a procedure is suggested to estimate them directly in the solution algorithm of the inverse problem.

The results of numerical experiments and the interpretation of the data from SHF radiometric field measurements show that the suggested methods enable one to effectively suppress the errors due to nonlinearity, the presence of the cosmic background, and also calibration errors.

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