OPTICAL CHARACTERISTICS OF HEAT-MASS-HALO OF A CARBON PARTICLES EVAPORATING IN A LASER FIELD

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The efficiency factors of light absorption and scattering by a halo of secondary particles produced by evaporation of an initial carbon particle in a field of high-power laser radiation have been calculated. The laser radiation is shown to be significantly attenuated because of scattering by heat halo and absorption by secondary condensed aerosol produced by evaporation of the initial particle.

We will consider evaporation of a carbon particle suspended in air in a field of high-power laser radiation in a subsonic regime. As is shown in Ref. 1, a cloud of submicron aerosol is formed in the vicinity of particle. An induced heat-mass-halo may result in excess attenuation of electromagnetic radiation. In the present paper, cross section of light attenuation by an evaporating carbon particle of coarsely dispersed fraction is studied.

Let us introduce the effective refractive index of a medium perturbed in the vicinity of the evaporating particle

$$n_{\text{eff}} = n_{\text{g}} + n_d + in'_d \, , \tag{1}$$

where

$$n_{\text{g}} = 1 + 2\pi \sum_{k=1}^{2} P_k(r) \frac{\beta_k}{k_{\text{B}}\, T(r)}$$

is the refractive index of a vapor-gas mixture,

$$n_d + in'_d = 2\pi\alpha_d \, N(r)$$

is the additional term describing optical properties of condensed aerosol. Here P_k and β_k are the partial pressure and polarizability coefficient of the kth component of a vapor-gas mixture, respectively ($k = 1$ is for the carbon vapor and $k = 2$ is for the air); k_{B} is Boltzmann's constant; $T(r)$ is the temperature of a medium at the distance r from the center of the evaporating particle; $\alpha_d = a^3(m^2 - 1)/(m^2 + 1)$ is the polarizability of a condensed spherical particle with radius a and complex refractive index m; $N(r)$ is the number density of condensed aerosol.

In the Van de Hulst approximation valid for the diffraction parameter of heat-mass-halo $\rho_d \gg 1$ and $|n_{\text{eff}} - 1| \ll 1$, the expression for the efficiency factor of light absorption by the condensed aerosol under condition of small run-on of the wave phase has the form[2]

$$K_{\text{h}}^{\text{ab}} = 2B^2 \int_0^{\pi/2} 4\rho_d \, n'_d(\tau) \cos\tau \sin\tau \, \mathrm{d}(\cos\tau) \, , \tag{2}$$

where $\tau = \arccos(x/B)$, $B = R_{\text{h}}^{\text{cl}}/R$ is the dimensionless radius of a halo formed by condensed aerosol, x is the dimensionless radial distance; $\rho_d = 2\pi R_{\text{h}}^{\text{cl}}/\lambda$, λ is the light wavelength, and R is the radius of evaporating particle. The estimation of the run-on of the phase gives the value of $\Delta\varphi \sim 0.15$ for $m = 1.96 - i\,1.01$, $a \sim 5{\cdot}10^{-9}$ m, $N = 10^{20}$ m^{-3}, $\beta_2 = 1.7{\cdot}10^{-30}$ m^3, $P_2 \sim 10^5$ N/m^3, characteristic radius of a heat halo $R_{\text{h}}^{\text{heat}} \sim 50\,R$, and radius of the cloud of secondary particles $R_{\text{h}}^{\text{cl}} \sim 20R$; hence, approximation (2) is valid.

Let us consider the estimation for a dimensionless absorption cross section that can be used to obtain the preliminary information on K_{h}^{ab} without cumbersome calculations. As is shown in Ref. 1, the vapor condensation occurs at the distance $r \sim (2\text{–}4)\,R$. Hence, the absorption within the zone of phase transition can be neglected in comparison with that by the whole condensed aerosol cloud. Then for calculation of K_{h}^{ab} it is suffice to find the number density of aerosols outside of the condensation zone, where the aerosol number density is described by the equation

$$\frac{\partial N(t, r)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \, V(t, r) \, N(t, r)\right) = 0 \tag{3}$$

with the boundary conditions

$$N(t = 0, r) = 0, \;\; N(t, r = R_1) = N_{\text{s}}(t),$$

where $V(t, r)$ is the velocity of the Stefan flow, R_1 is the external radius of the zone of phase transition, and $N_{\text{s}}(t)$ is the number density of secondary (condensed) particles at $r = R_1$. Making the substitution of variable $N_0 = N/\rho(r)$, where $\rho(r)$ is the concentration of heterogeneous mixture (total number of carbon vapor atoms, condensed matter atoms, and air molecules per unit volume), and using the Laplace transform with allowance for weak dependence of $\rho(t, r)$ and $V(t, r)$ on time in comparison with r, the solution of equation (3) will be written in the form

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$$N(t, r) = \begin{cases} \dfrac{N_s(t)\, T(R_1)}{T(R)}, & \text{at } t \le \displaystyle\int_R^r \frac{dr}{V(r)}, \\ 0, & \text{at } t > \displaystyle\int_R^r \frac{dr}{V(r)}. \end{cases} \quad (4)$$

Outside of the condensation zone ($r \ge (2–4)R$) the temperature of a medium is less than 3000 K. This permits us to use the analytical dependence of the coefficient of thermal conductivity $\varkappa(T)$ on temperature[3] in the equation for temperature field in the vicinity of evaporating particle in the absence of condensation[4]

$$T(x) = T_0(1 + c_0(p + 1)/x)^{1/(p+1)}, \quad (5)$$

where $c_0 = \displaystyle\int_{T_0}^{T_s} \frac{\varkappa(\xi)d\xi}{\varkappa(T_0)T_0}$, $p = 0.75$ (see Ref. 3), and T_0 is the temperature of unperturbed medium. By substituting in Eq. (2) the derived expression for the number density $N(t, r)$, accounting for Eq. (5), we find

$$K_h^{ab} = \frac{8a_1\pi}{\lambda} R(t)\, n_d'(T_0) \left(\frac{B_0}{c_0(p + 1)}\right)^{\frac{1}{p+1}} B^3, \quad (6)$$

where

$$a_1 = \frac{1}{1 + b} - \sum_{k=1}^{\$} \frac{n(n - 1)\,...\,(n - k)(n - k + 1)}{(k + 1 + b)k!},$$

$$n = \frac{1}{2}, \quad b = \frac{1}{2(p + 1)},$$

$$n_d'(T_0) = 2\pi\, \mathrm{Im}(\alpha_d)\, N_s(t) \frac{T(R_1)}{T_0}.$$

Dimensionless radius of the halo of secondary particles can be found using the mass conservation law

$$\frac{4}{3}\pi\rho_{car}(R_0^3 - R^3(t)) = \int_R^{R_h^{cl}} \left(m_0\rho_1(r) + \frac{4}{3}\pi\rho_{car}\, a^3 N(t, r)\right) 4\pi r^2\, dr. \quad (7)$$

Here m_0 is the carbon atomic mass, $\rho_1(r)$ is the concentration of the carbon vapors, ρ_{car} is the carbon density, and a and R_0 are the radii of secondary and initial particle, respectively. Applying the Bonnet formula[5] to the right-hand side of Eq. (7) we obtain

$$\frac{4}{3}\pi\rho_{car}(R_0^3 - R^3(t)) = \frac{4}{3}\pi R_h^{cl} f(R_h^{cl}) + \frac{4}{3}\pi\xi^3 (f(R) - f(R_h^{cl})) - \frac{4}{3}\pi R^3 f(R),$$

where

$$f(r) = \frac{4}{3}\pi\rho_{car}\, a^3 N(r) + m_0\rho_1(r), \quad \xi \in [R, R_h^{cl}].$$

The function $f(r)$ on a particle surface and on the external boundary of the halo of secondary particles takes the following values: at $r = R_h^{cl}$ all vapor already has been condensed and in this case $f(R_h^{cl}) = \frac{4}{3}\pi\rho_{car}\, a^3 N(R_h^{cl})$. The condensation is absent on the surface of a particle; therefore, at $r = R(t)$, $f(R) = m_0\rho_1(R)$. Then we obtain that dimensionless radius of the halo $B = R_h^{cl}/R$ satisfies the relation

$$B^3 = \frac{\left[1 - \left(\frac{R(t)}{R_0}\right)^3\right] T(B)}{(4/3)\,\pi\, a^3 N_s(t, R_1)\, T(R_1/R)}, \quad (8)$$

which holds for $R_0^3 f(R_0) \ll R_h^3 f(R_h^c)$. Solving implicit equation (8), we may find the dependence of the halo radius B on the radius of the evaporating particle $R(t)$, hence, on the time t. Using expression (8), let us estimate the maximum halo radius reached at $R(t) = 0$ (particle has been evaporated completely)

$$B_{max}^3 = \frac{3}{4\,\pi a^3 N_s}.$$

For $N_s \sim 10^{20}\ \mathrm{m}^{-3}$ and $a \sim 5\cdot10^{-9}$ m the radius of a halo reaches maximum $B_{max} \simeq 20$. Combined calculation by formulas (6) and (8) permits particular values for the efficiency factor of absorption K_h^{ab} to be obtained. We present numerical values of some parameters that are used in calculation. So, for the complex refractive index $m = 1.96 - i1.01$ and radius of a secondary particle $a = 5\cdot10^{-9}$ m, the imaginary part of polarizability is equal to $\mathrm{Im}(\alpha_d) = 3.75\cdot10^{-26}\ \mathrm{m}^3$. At $R_1 = 4R$, the temperature of a medium at the point in which the phase transitions terminate, is equal to $T(R_1) = 3000$ K, the number density of secondary particles $N_s(R_1) \sim 10^{20}\ \mathrm{m}^{-3}$. For the laser wavelength $\lambda = 10.6\ \mu$m, $R_0 = 100\ \mu$m, and $c_0(T_{sur} = 4500\ \mathrm{K}) = 190$, the efficiency factor of absorption, according to Eq. (6) is equal to

$$K_h^{ab}(B = 9) = 3.2, \quad K_h^{ab}(B = 10) = 4.7,$$

i.e., it is comparable to the factor of light attenuation by a large particle.

In Fig. 1 the time dependence of dimensionless cross-section of light absorption by a halo of secondary particles is shown for comparison obtained on the basis of numerical solution of a system of equations describing the processes of formation of secondary particles in the vicinity of evaporating particle.[1]

After a short time from the beginning of evaporation (characteristic time of evaporation $t \sim 3\cdot10^{-1}$ s), the absorption efficiency factor reaches fast the value comparable to the efficiency factor of attenuation by a large particle. This is connected with fast reduction of mass ($\Delta m \sim 4\rho_{car}\,\pi R^2 dR$) and formation of secondary particles.

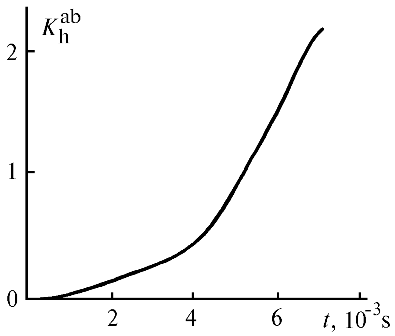

FIG. 1. Time dependence of the efficiency factor of light absorption by a halo of secondary particles for $I_0 = 2\cdot 10^8$ W/m^2, $\lambda = 10.6$ μm, *and* $R_0 = 100$ μm.

Let us proceed to consideration of the efficiency factor of light scattering by a heat-mass-halo K_h^{sc}. In the Rayleigh-Hans approximation for a medium with the complex refractive index $m = n_g + n_d + in'_d$ the factor K_h^{sc} is determined by the formula[2]

$$K_h^{sc} = \frac{k^2 R^2}{4} \int_0^{\pi} (1 + \cos^2\theta)\, d\cos\theta \times$$

$$\times \left| \int_1^A 2(n_g(z) + n_d - 1 + in'_d) z \frac{\sin\left(2kRz \sin\frac{\theta}{2}\right)}{\sin\frac{\theta}{2}}\, dz \right|^2 . \quad (10)$$

Here, we took into account that $|n_g - 1| \ll 1$, $n_d \ll 1$. Using the dependence of temperature of a medium on the dimensionless radial distance $z = r/R$ for expressions $n_d(T(z))$ and $n'_d(T(z))$ and integrating Eq. (9), we obtain the estimation from above for the efficiency factor of scattering

$$K_h^{sc} \le 4(n_g(T_s) - 1)^2 \frac{(kR)^2}{2} A^4 + 4(n^2(R_1) + n'^2_d(R_1)) \times$$

$$\times \frac{kR^2}{2} B^4 \left(\frac{B}{c_0}\right)^{\frac{2}{p+1}} + 4(n_g(T_{sur}) - 1) n_d(R_1) \left(\frac{B}{c_0}\right)^{\frac{1}{p+1}} . \quad (11)$$

Here, A is the dimensionless radius of a heat halo, $k = 2\pi/\lambda$ is the wave number; T_{sur} is the surface temperature of evaporating particle. The first term in Eq. (11) is the efficiency factor of light scattering by nonuniformly heated vapor-gas mixture, second term is the efficiency factor of light scattering by secondary aerosol. The third term is caused by interaction between scattering media. We designate three terms in Eq. (11) by K_{sc}^{heat}, K_{sc}^{sec}, and K_{sc}^{int}, respectively. Let us consider the ratio of the efficiency factor of scattering by secondary particles to that of absorption by these particles

$$\frac{K_{sc}^{sec}}{K_h^{ab}} = 2kRB \left(\frac{B}{c_0}(1 + p)\right)^{\frac{1}{p+1}} n_d(R_1)\, T_0 T(R_1)\, a_1 . \quad (12)$$

For $B = B_{max} = 20$, $R = 100$ μm, $\lambda = 10.6$ μm, $p = 0.75$, $n_d(R_1) = 5\cdot 10^{-5}$, $a_1 = 0.58$, and $T(R_1) = 3000$ K ratio (12) assumes the value $\frac{K_{sc}^{sec}}{K_h^{ab}} \simeq 0.01$, which indicates predominant attenuation of laser radiation due to its absorption by secondary particles. Let us estimate the ratio of the efficiency factor of light scattering by heat halo to that of light absorption by secondary particles

$$\frac{K_{sc}^{heat}}{K_h^{ab}} = \frac{(n_g(T_{sur}) - 1)^2\, kRA^4}{2an'_d(T_0)\, B^3} \left(\frac{c_0(p + 1)}{B}\right)^{\frac{1}{p+1}} .$$

At the surface temperature of evaporating particle $T_{sur} = 4500$ K, laser radiation wavelength $\lambda = 10.6$ μm, $B_{max} = 20$, $A = A_{max} = 45$, $R = 100$ μm, $4(n_g - 1)^2 = 5\cdot 10^{-9}$, and $n'_d(T_0) \sim 2.5\cdot 10^{-4}$ the ratio $\frac{K_{sc}^{heat}}{K_h^{ab}} \sim 2$.

Thus, the above-presented calculations have shown that it is necessary to take into account the contribution of light scattering by a heat halo and absorption by condensed aerosol produced due to evaporation of carbon particles to attenuation of laser radiation.

In Fig. 2, the dependence of the efficiency factor of light scattering by a thermal halo on the halo radius calculated on the basis of expression (11) is shown.

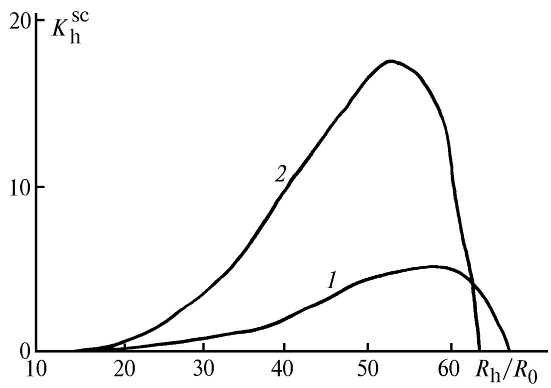

FIG. 2. Dependence of the efficiency factor of light scattering by a heat halo on the halo radius at $T = 4500$ K, $R_0 = 5$ (*1*) *and* 10 μm (*2*).

REFERENCES

1. V.I. Bukatyi and T.K. Kronberg, Atmos. Oceanic Opt. **7**, No. 9, 671–674 (1994).
2. H.C. van de Hulst, *Light Scattering by Small Particles* (Willey, New York, 1957).
3. V.K. Pustovalov and I.A. Khorunzhii, in: *Effect of Intense Laser Radiation onto Solid Aerosol. Interinstitute Scientific Paper Collection* (Barnaul, 1987), pp. 68–80.
4. V.I. Bukatyi, V.N. Krasnopevtsev, and A.M. Shaiduk, Fiz. Goreniya i Vzryva, No. 1, 41–48 (1988).
5. G.M. Fikhtengolts, *Course of Differential and Integral Calculus* (Nauka, Moscow, 1970), Vol. 1, 608 pp.