Effects of quasi-Gaussian distribution of the sea-surface slopes at laser sounding

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Analysis presented in this paper used the data of field measurements. It has been found that deviation of the slope distribution from the Gaussian distribution results in an increase in the probability of spot-like specular reflections at laser sounding along nadir. It is shown that the probability of specular reflection as a function of wind velocity is mainly determined by variation of the mirror points' density and depends, to a lesser degree, on the slope dispersion.

Introduction

Until so far all the models of light scattering by the sea surface have presented the latter in the form of a random moving Gaussian surface.[1] Such an approach is employed both in analyzing scattering[2] of solar radiation and in interpreting the results of laser sounding.[3]

At the same time, a series of field experiments have revealed that the sea surface is not strictly Gaussian. In the distribution of the sea surface slopes deviations from the Gaussian distribution are observed, which increase with the wind growing higher.[4–6] This results in variation in the characteristics of the ocean–atmosphere interface characterizing it as a reflecting surface. In particular, the variation of asymmetry of the longitudinal slope component with the increase of wind velocity is manifested in the angular shift of the light reflection maximum.[7]

The goal of this paper is to evaluate the influence of statistical deviations of the sea surface slopes from the Gaussian distribution on the return signal characteristics at laser sounding along nadir.

Two-dimensional model of quasi-Gaussian surface

We introduce the Cartesian coordinates. One of the axes of this coordinate system is oriented in the direction coincident with the main direction of wave propagation, and the other one is oriented along the orthogonal direction. Let us denote the surface deviation by ξ and assume that $d\xi/dx = \xi_u$ and $d\xi/dy = \xi_c$ are the longitudinal and transverse components of the slope. Then the slope components and its modulus ξ_m (or the overall slope) are determined by the expressions

$$\xi_u = \xi_m \cos\theta;\ \xi_c = \xi_m \sin\theta;\ \xi_m = \sqrt{\xi_u^2 + \xi_c^2},$$

where θ is the slope direction. Furthermore, for all the characteristics of slopes the index u corresponds to the longitudinal component and the index c corresponds to the transverse component, the index m corresponds to the slope modulus. Now we introduce the following designation: σ^2 is the parameter variance, which is denoted by the subscript.

Let us denote the probability densities of the slope components as $P_u(\xi_u)$ and $P_c(\xi_c)$. Because the longitudinal and transverse slope components do not correlate, the two-dimensional probability density of slope component equals

$$P_{2D}(\xi_u, \xi_c) = P_u(\xi_u)\, P_c(\xi_c).$$

Let us turn from the Cartesian coordinates to polar coordinate system

$$P_p(\xi_m,\theta) = \left|\frac{\partial(\xi_u,\xi_c)}{\partial(\xi_m,\theta)}\right| P_{2D}(\xi_u,\xi_c), \quad (1)$$

where the Jacobian $\left|\frac{\partial(\xi_u,\xi_c)}{\partial(\xi_m,\theta)}\right| = \xi_m$. In integrating $P_p(\xi_m, \theta)$ over all directions we obtain the statistical distribution of the slope modulus,

$$P_m(\xi_m) = \int_0^{2\pi} P_p(\xi_m,\theta)\, d\theta . \quad (2)$$

The surface characteristics, when the slope components obey the normal distribution law, have been studied in the literature in detail.[1] It is shown that in this case the type of the slope modulus distribution determines one parameter, the three-dimensionality index, $\gamma = \sigma_c/\sigma_u$, where σ_u^2 and σ_c^2 are the slope component variances.

To take into account the observed deviations of the slope distributions from the Gaussian distribution the models are commonly used, in which the probability density, $P_u(\xi_u)$ and $P_c(\xi_c)$ are approximated by the Gram–Charlier series.[4,5] The coefficients of this series are calculated by the empirical estimates of the slope distribution moments.

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Traditionally, in the experiments the moments are determined up to the fourth order inclusive. Therefore, in practice, the approximation of slope distribution is limited by the first five terms of the Gram–Charlier series. If we introduce the normalization $\tilde{x} = x/\sigma_x$ and take into account that the mean slope in any direction equals zero, then the slope component distribution can be written in the form

$$P_{\text{G-C}}(\tilde{x}) = P_{\text{N}}(\tilde{x}) \times$$
$$\times\left[1+\frac{1}{6}\mu_3 H_3(\tilde{x})+\frac{1}{24}(\mu_4-3)H_4(\tilde{x})\right], \qquad (3)$$

where P_{N} is the normal distribution; μ_i is the ith statistical moment; H_i is the orthogonal Chebyshev–Hermite polynomial of the ith order. In this case $\mu_3 = A$ is the distribution asymmetry, $\mu_4 - 3 = E$ is the excess.

Replacing the Gram-Charlier series by the approximation (3) may result in negative values of the probability density on the “wings” of the distribution.[8] However, when sounding along nadir the light spots arrive at the receiver aperture, which are formed by small areas with the orientation close to the horizontal one. The slope of small area must not exceed the critical angle ξ_{cr}. The value ξ_{cr} is determined by the geometric dimensions of the receiver aperture and the distance from it to the reflecting surface.[9] Field measurements, as a rule, are carried out at very small critical angles. Therefore, the use of the approximation (3) here is quite correct.

Relationship between light spot characteristics and the sea-surface slope characteristics

The probability of recording the light spot at laser sounding along nadir

$$\delta = \int_0^{\xi_{\text{cr}}} P_{\text{m}}(\xi_{\text{m}})\,\text{d}\xi_{\text{m}} \qquad (4)$$

is determined by two factors, namely, the distribution shape $P_{\text{m}}(\xi_{\text{m}})$ and the variance (or mean value) of the slope modulus. Let us consider these factors separately.

As a criterion, determining the influence of deviations of real distribution of slope components from the Gaussian distribution, the following parameter

$$\varepsilon_1 = \delta \Big/ \int_0^{\xi_{\text{cr}}} P_{\text{mN}}(\xi_{\text{m}})\,\text{d}\xi_{\text{m}}, \qquad (5)$$

is used, where $P_{\text{mN}}(\xi_{\text{m}})$ is the slope modulus distribution in the situation when its components are distributed normally.

In Eq. (3) the Chebyshev–Hermite polynomial of the third order of magnitude $H_3(\tilde{x}) = \tilde{x}^3 - 3\tilde{x}$ reduces to zero at $\tilde{x} = 0$. Therefore at sounding along nadir, in case of small values of ξ_{cr}, the influence of asymmetry of the longitudinal component of sea surface slopes on the value ε_1 should be small. This value must be determined mainly by the excesses in distributions of the longitudinal and transverse slope components.

To calculate the value of the parameter ε_1, we use the estimates of statistical moments of slope components obtained during three experiments.[4–6] Mean values of E_c in these experiments are closely related and it is assumed that $E_c = 0.4$. For E_u two values, 0.23 and 0.4, were obtained. The calculations of ε_1, in the case when $\gamma = 1$, gave the following results: at $E_u = 0.23$ and $E_c = 0.4$ the parameter $\varepsilon_1 = 1.08$; at $E_u = 0.4$ and $E_c = 0.4$ the parameter $\varepsilon_1 = 1.10$.

Now we consider the situation of anisotropic slopes, $\gamma < 1$. In literature[4–6] the inverse values of the square of the three-dimensionality index γ can be found. Based on these values, the mean value γ for three experiments can be taken to be 0.81. It turned out that because the effect of anisotropy influences equally as the probability δ in the case of Gaussian and quasi-Gaussian surfaces, the relative probability ε_1 remains practically constant in changing the parameter γ.

We now turn to analysis of the second factor, i.e., the variation of the level of sea surface roughness. It is believed that the shape of slope component distributions with the increase of the wind velocity W does not vary. In particular, this indicates that the function $P_{\text{m}}(\xi_{\text{m}}/\bar{\xi}_{\text{m}})$ does not vary at changing the mean value of the slope modulus $\bar{\xi}_{\text{m}}$.

Taking into account $\xi_{\text{cr}} \ll \bar{\xi}_{\text{m}}$, the function $P_{\text{m}}(\xi_{\text{m}}/\bar{\xi}_{\text{m}})$, near the zero value of the slope modulus, can be approximated by the linear dependence. Then the probability δ_2 of the fact that the value of slope modulus satisfies the condition $0 \le \xi_{\text{m}} \le \xi_{\text{cr}}$, is proportional to the ratio $[\xi_{\text{cr}}/\bar{\xi}_{\text{m}}(W)^2]$.

Using the empirical dependence[10] of the mean value of the slope modulus on the wind velocity $\bar{\xi}_{\text{m}} = 6.9 \cdot 10^{-3}\, W + 0.129$ we obtain

$$\delta_2(W) \sim (6.9 \cdot 10^{-3}\, W + 0.129)^{-2}. \qquad (6)$$

It should be noted that the expression (6) is valid for the case when the dimensions of a laser spot on the surface are much less than the local radius of the sea-surface curvature, i.e., when the variation of the slope inside the laser spot can be neglected. If the spot size is large, it is necessary to consider one more factor – the increase in the density of the points of specular reflection on the sea surface. This effect must result in the growth of the probability of specular reflection. At a reasonably large spot the situation will appear when, starting from a certain wind velocity, within its limits at last one point of specular reflection will be found.

Having introduced the function $R(W)$, describing the increase of the specular reflection point density with the growth of wind velocity W, we obtain

$$\delta_3(W) = \delta_2(W)\, R(W). \qquad (7)$$

The type of the function $R(W)$ is determined below based on the experimental data.

Comparison with laser sounding data

For analysis we use results of the experiments on laser sounding of sea surface carried out at the oceanographic platform of the Marine Hydrophysical Institute. The optical layout of the instrument for laser sounding and the conditions of measurements have been described in Ref. 9. The instrument was used with a combined source and a receiver of optical signal. The diameter of the receiver aperture was equal to 5 cm. The sounding was performed along nadir at 5 m height. For a given receiver and the conditions of measurements selected $\xi_{cr} = 0.0025$.

The spot diameter at the unperturbed surface was 5 mm. This linear size exceeds the typical wavelength scale, separating the gravitation-capillary and capillary ranges, which equals 4 mm. Therefore it is expected that at medium and high winds the surface slope will greatly change within the light spot.

According to the laser sounding data the glare probability can be determined as

$$\delta_{meas}(W) = \tau(W)/T, \quad (8)$$

where τ is the total duration of glares recorded during the measurement run of duration T. We compare how the empirical parameter δ_{meas} and the model probability determined only by the mean value of the slope modulus and the density of specularly reflecting points change with the variation of wind velocity. In our experiment the density of these points was not determined. Therefore we use the fact that the variation of the density of specularly reflecting points is proportional to the variation of the frequency F of recorded glares.

Let us consider the relative variations of probabilities $\varepsilon_{meas}(W)=\delta_{meas}(W)/\delta_{meas}(W_0)$, $\varepsilon_2(W)=\delta_2(W)/\delta_2(W_0)$, and $\varepsilon_3(W)=\delta_3(W)/\delta_3(W_0)$. We assume that $R(W)=F(W)/F(W_0)$. For ε_2 the normalizing factor is determined at $W_0 = 8$ m/s. For the remaining parameters the normalizing factor was determined by the data of laser sounding as the mean value for the corresponding parameter at $7.5 < W < 8.5$ m/s. The above-mentioned range of wind velocity values was chosen because at gentle winds a large spread of slope statistics was observed, and at values W higher than this range the effects of decrease of glare intensity below the threshold of the receiver sensitivity began to appear.

The increase of $\bar{\xi}_m$ values and the growth of glare density result in the opposite effects (Fig. 1). With increasing $\bar{\xi}_m$ the probability of glare recording decreases monotonically. Variation of the value of $\bar{\xi}_m$ from the values, corresponding to the calm conditions, to the values corresponding to $W = 15$ m/s results in a decrease of the parameter ε_2 by a factor of three.

$R(W)$ varies within wide limits. In our experiment the growth of the F parameter was observed when the wind grew from the values of calm to $W \approx 10$ m/s (Fig. 1). The variation of the behavior of $F = F(W)$ at $W > 10$ m/s is due to the increase of the local surface curvature resulting in the decrease in the lidar return intensity. Therefore, some glares, entering the receiver aperture, are not recorded, since the glare intensity is found to be lower than the detector's threshold sensitivity.[9]

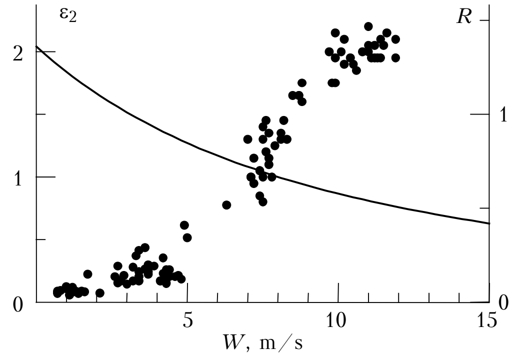

Fig. 1. Dependences of the parameters R (points) and ε_2 (solid line) on the wind velocity W.

A comparison of the dependences on wind velocity of the parameters R and ε_2 shows that the effect, related to the growth of the specular reflecting points' density, dominates over the effect due to the increase of $\bar{\xi}_m$. This effect mainly determines the type of the function $\varepsilon_3 = \varepsilon_3(W)$.

Now we compare how parameters ε_3 and ε_{meas} change with the variation of wind velocity. To calculate the values of the parameter ε_{meas}, we took the same array of laser sounding data, which was used to calculate the values of the parameter R. On the whole the behavior of the parameters ε_3 and ε_{meas} in the entire range of wind velocities is similar (Fig. 2).

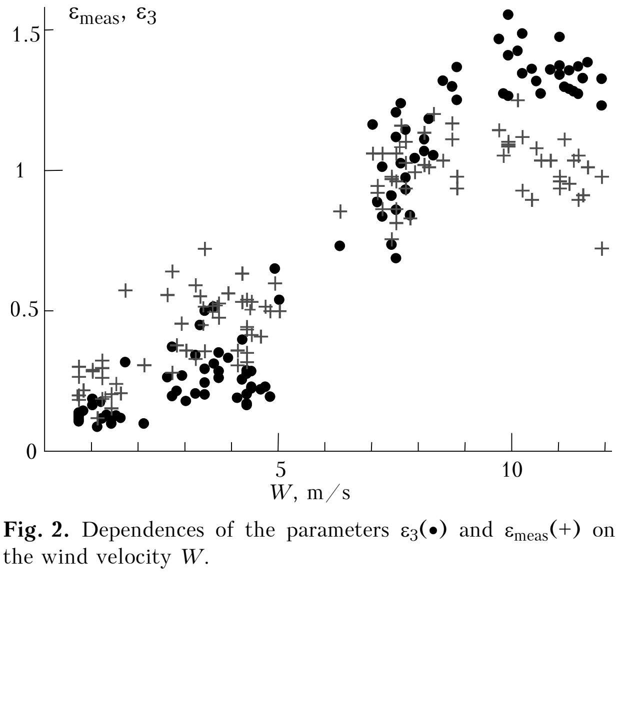

Fig. 2. Dependences of the parameters ε_3(•) and ε_{meas}(+) on the wind velocity W.

However, the parameter ε_{meas} varies within more narrow limits. In particular, this can be related to the fact that in this case the decrease of local radii of the surface curvature is not taken into account.

Conclusions

The factors were analyzed, which affected the probability of recording the spots of specular reflection at laser sounding along nadir. It is shown that the deviations of slope component distributions from the normal law, observed in the field experiments, must result in the increase of the probability of the spots of specular reflection at laser sounding along nadir. This is caused by the fact that the values of excess of slope components exceed the value corresponding to the normal law. Since the coefficients E_u and E_c do not depend on the wind velocity, this effect is a factor reducing the accuracy of the problem solution for determining the wind velocity using laser sounding data.

Under conditions of growing wind, we observe the growth of the probability of light spots of specular reflection. The growth occurs until a substantial part of flares falls to be below the threshold of the photodetector sensitivity because of the increase of local surface curvature and the corresponding decrease of the glare intensity. It is established that from two factors affecting the probability of light spots of specular reflection, namely, the variation of the specular reflection spot density and the variation of slope variance, the first factor dominates.

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