ON THE DETERMINATION OF THE ATMOSPHERIC OPTICAL DEPTH DUE TO AEROSOL SCATTERING FROM SKY BRIGHTNESS MEASUREMENTS IN THE VISIBLE REGION OF THE SPECTRUM

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Results of the numerical solution of the radiative transfer equation for the atmosphere with a trimodal particle size distribution are analyzed. Approximate formulas are derived by which the aerosol optical depth can be determined from observations of the absolute phase functions of the sky brightness.

In the solution of many problems of actinometry and atmospheric optics, it is often necessary to operationally determine the optical depth of the atmosphere τ and its components due to light scattering and absorption under daytime conditions. This need is dictated, to a certain extent, by the ubiquitous increase of the air turbidity. The total optical depth τ outside the molecular absorption bands can be determined from measurements of direct solar radiation intensity using the Bouguer method. However, after separating out the Rayleigh component τ_R, subsequent separation into the components

$$\tau = \tau_R + \tau_a + \tau_n, \tag{1}$$

where τ_d and τ_a are the optical depths due to aerosol scattering and absorption, respectively, is not possible without additional information on the scattering properties of the atmosphere. Data from observations of the absolute brightness phase function $f(\varphi)$ for a clear sky in the solar almucantar (φ is the scattering angle) may serve as a source of such information. In this paper the radiative transfer equation is used to analyze a rather simple method, which was suggested earlier, for determining τ_d on the basis of measurements of $f(\varphi)$ (Ref. 1). Also, some recommendations as to its practical application in the visible region of the spectrum are made.

Following Pyaskovskaya-Fesenkova,[2] we represent the experimental absolute brightness phase function in the form

$$f(\varphi) = f_R(\varphi) + f_d(\varphi) + f_{2q}(\varphi), \tag{2}$$

where $f_R(\varphi)$ and $f_d(\varphi)$ are the molecular and aerosol single-scattering phase functions, respectively, and $f_{2q}(\varphi)$ is the additional term due to multiple scattering and light reflection from the underlying surface with albedo q. Assuming that the light flux single-scattered by the aerosol particles into the backward hemisphere is approximately equal to the difference of the fluxes of multiple-scattered radiation into the forward and backward hemispheres, i.e.

$$2\pi \int_{\pi/2}^{\pi} f_d(\varphi) \sin\varphi \, d\varphi \approx 2\pi \int_0^{\pi/2} f_{2q}(\varphi) \sin\varphi \, d\varphi - 2\pi \int_{\pi/2}^{\pi} f_{2q}(\varphi) \sin\varphi \, d\varphi, \tag{3}$$

it is quite easy to arrive at the following ratio[1]:

$$\tau_d \approx \tau'_d = 2\pi \int_0^{\pi/2} f(\varphi) \sin\varphi \, d\varphi - 2\pi \int_{\pi/2}^{\pi} f(\varphi) \sin\varphi \, d\varphi, \tag{4}$$

enabling us to determine the aerosol scattering optical depth τ_d from the observations of $f(\varphi)$. This method has a number of practical advantages as compared to other techniques,[2] since, e.g., the thusly determined τ_d values do not depend on the aerosol atmospheric stratification,[3] or on the presence of a weak absorption,[1,2] or on the albedo of the underlying surface,[1] if the latter is close to lambertian. The question of the validity of assumption (3) and, hence, of expression (4) can be answered only theoretically by calculating $f(\varphi)$ for a reasonably chosen shape of the aerosol scattering phase function $f_d(\varphi)$.

In the construction of the aerosol model for calculating $f(\varphi)$ at different wavelengths and different values of the atmospheric turbidity, it was hoped that such a model would adequately describe real optical properties of the atmosphere. According to Ref. 5, the optical characteristics of atmospheric haze are reliably described by three lognormal particle size distributions including the Aitken nuclei (a), the submicron (b), and the coarse aerosol fractions (c). The choice of the distribution parameters and the weight relations between the fractions can be made using the mean aerosol scattering function[6] found from observations in the spectral region $\lambda = 0.55$ μm. Using the tables from Ref. 7, it was found that this function in the range of scattering angles $2° \le \varphi \le 160°$ observable in practice was best approximated by the sum of three phase functions corresponding to the following distribution parameters $\sigma^2 = 0.3$ and $\alpha' = -1.0$ (the Aitken nuclei fraction, 15%), $\sigma^2 = 0.4$ and $\alpha' = 0.4$ (the submicron fraction, 60%), $\sigma^2 = 0.5$ and $\alpha' = 0.8$ (the coarse fraction, 25%) with the refractive index $n = 1.5$ (Fig. 1). Here, σ is the variance of the logarithm of the radius; $\alpha' = \ln\rho_0$; $\rho_0 = 2\pi r_0/\lambda$; and r_0 is the geometrical mean radius of the particles.

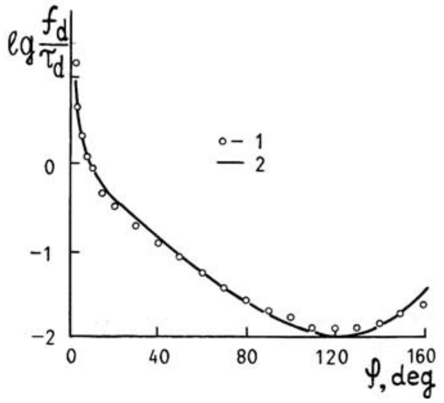

FIG. 1. Aerosol scattering phase function obtained from the observational data (1) and its approximation by a trimodal size distribution function (2).

The optical depths corresponding to each of the above-indicated groups, of particles, τ_{da}, τ_{db}, and τ_{dc}, are given in Table I, along with the total aerosol (τ_d) and Rayleigh (τ_R) optical depths. All the aerosol depths correspond to the conditions of high atmospheric transmissivity for the turbidity factor $T = (\tau_R + \tau_d)/\tau_R$ in the spectral region $\lambda = 0.55$ μm equal to 2. In this paper the cases $T = 3$ and $T = 4$ corresponding to moderate and strong atmospheric turbidity, respectively, are also considered.

TABLE I

λ, μm	τ_{da}	λ_{db}	τ_{dc}	τ_d	τ_R
0.40	0.0314	0.0599	0.0231	0.1144	0.3631
0.55	0.0147	0.0588	0.0245	0.0980	0.0980
0.65	0.0096	0.0549	0.0249	0.0894	0.0500

The calculations of $f(\varphi)$ are performed by solving the radiation transfer equation numerically for the plane-parallel model of the atmosphere:

$$\mu\frac{dT}{dt} + I + \frac{\omega}{4\pi}\int_0^{2\pi} d\psi' \int_{-1}^{1} g(\varphi') I(t,\mu',\psi') d\mu' + \frac{\omega}{4} S g(\varphi) e^{-t/\zeta};$$

$$I(0,\mu,\psi) = 0, \quad \mu > 0; \tag{5}$$

$$I(\tau,\mu,\psi) = \frac{q}{4\pi}\int_{4\pi}^{2\pi} \partial\psi' \int_{-1}^{1} I(\tau,\ \mu',\ \psi')\mu' d\mu' + qSe^{-\tau/\zeta},$$

$$\mu < 0,$$

where $I(t,\ \mu,\ \psi)$ is the intensity of scattered radiation at the optical depth t; $\mu = \cos\theta$ and ψ are the spherical coordinates of the direction of scattering; $g(\varphi)$ is the single-scattering phase function, which is related to $f_1(\varphi)$ by the relation

$$g(\varphi) = \frac{4\pi}{\tau} f_1(\varphi); \tag{6}$$

ω is the single-scattering albedo (in our case, we set $\omega = 1$); $\zeta = \cos Z_0$; Z_0 is the solar zenith angle; πS is the spectral solar constant. To solve Eq. (5), we used the method of spherical harmonics in a modification[8] which provides efficient numerical solution of radiation transfer problems in media with high anisotropy of scattering. Using this algorithm, we have calculated the diffuse radiation intensities in the solar almucantar $I(\tau,\ \zeta,\ \psi)$ connected with the brightness phase function by the following relation

$$f(\varphi) = \frac{I(\tau,\zeta,\psi)}{\pi S\ \sec Z_0 \exp(-\tau\ \sec Z_0)}, \tag{7}$$

where

$$\cos\varphi = \zeta^2 + (1 - \zeta^2)\cos\psi. \tag{8}$$

The latter relation limits the possibilities of determining the function $f(\varphi)$ in the region of large scattering angles. At $\psi = 180°$, the maximum scattering angle $\varphi_{max} = 2Z_0$. Normally, the plane-parallel approximation of the atmosphere is considered to be acceptable if $\cos Z_0 \ge 0.2$, which corresponds to $\varphi_{max} \sim 157°$. If $Z_0 \ge 45°$, then the φ_{max} value does not exceed 90°, and the integrals in Eq. (4) cannot be calculated. The analysis of the .observations and the calculations of the functions $f(\varphi)$ show that this integral can be calculated with reasonable accuracy only if functions $f(\varphi)$ are available for which φ_{max} is not less than 120°. Thus, the practical use of relation (4) is limited to values of $\sec Z_0$ (or air mass) less than or equal to ~ 2–5 for reasons of purely geometric considerations and the applicability of the plane-parallel approximation.

In Fig. 2, the τ'_d values, found using formula (4), are plotted for different values of $\cos Z_0$ vs the true τ_d values assumed in the calculations of $f(\varphi)$.

A straight line is drawn at an angle of 45° to the horizontal axis. The vertical spread of points at each fixed value of τ_d is in close agreement with the decrease of cos Z. From Fig. 2 it can be seen that it is possible to use relation (4) to determine the aerosol scattering optical depths only for $\tau_d \sim 0.15$. At smaller τ_d, the use of formula (4) leads to an underestimate and at larger values, to an overestimate of the results. Analysis of the estimated data gives the following relation between the τ_d' and τ_d values

$$\tau_d \approx \tau_d'' = \frac{-(\delta+\beta\sec Z_0)+\sqrt{(\delta+\beta\sec Z_0)^2+4(\gamma+\varepsilon\cos Z_0)\tau_d'}}{2(\gamma+\varepsilon\sec Z_0)} \quad (9)$$

in Table II.

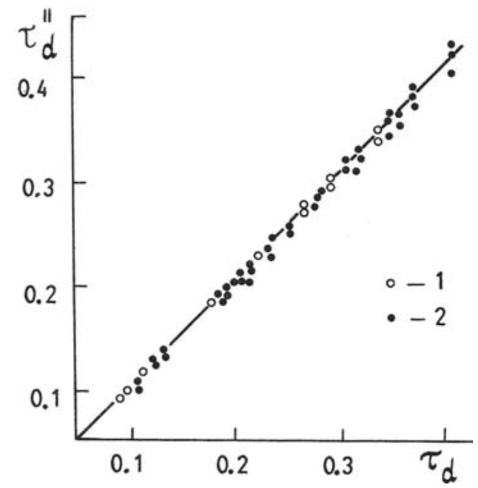

FIG. 2. The τ'_d values as a function of τ_d for 0.4 μm, *0.55* μm *(2), and 0.65* μm *(3).*

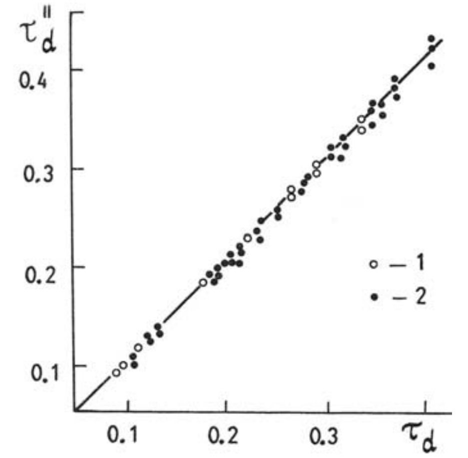

FIG. 3. The τ_d'' values corresponding to the initial data (1), and the additional (2) versions of the particle size distributions as a function of τ_d.

TABLE II

λ, μm	δ	β	γ	ε
0.40	0.835	–0.0035	0.392	0.260
0.55	0.822	–0.0040	0.320	0.310
0.65	0.800	–0.0100	0.187	0.320

The results of the comparison of the τ_d'' values calculated using expressions (4) and (9) with the τ_d values used in the calculations of $f(\varphi)$ are shown in Fig. 3.

In order to judge the prospects of expressions (4) and (9) for practical use, it is necessary to estimate the influence of the possible variations of the real scattering phase function on the proximity of τ_d'' and τ_d. Generally speaking, such variations were already included in the calculations of the dependence of the scattering phase function on the wavelength and the turbidity factor. Thus, the asymmetry coefficients of the scattered light fluxes

$$\Gamma_1 = \frac{\int_0^{\pi/2} f_1(\varphi)\sin\varphi\, d\varphi}{\int_{\pi/2}^{\pi} f_1(\varphi)\sin\varphi\, d\varphi} \quad (10)$$

take the extreme values 1.47 (Γ = 8.79) at T = 2 in the blue spectral region and 5.28 (Γ_d = 9.47) at T = 4 in the red spectral region. The corresponding asymmetry coefficients of the aerosol scattering phase function are given in brackets. Moreover, within each spectral range we have varied the aerosol scattering phase function by varying the aerosol particle fractions. Thus, the relative concentrations of the Aitken nuclei, the submicron particles, and the coarse fraction were varied from 15 to 25%, 60 to 70%, and 25 to 55%, respectively. The results of the τ_d'' calculations, taking into account such variations of the particle size spectra, are also presented in Fig. 3. These variations lead to a somewhat broader spread of the points without changing the overall picture. The rms deviations of τ_d'' from τ_d for all the considered cases at a confidence level of 0.95 are 2.8% (λ = 0.4 μm), 3.4% (λ = 0.55 μm), and 4% (λ = 0.65 μm).

Thus, relations (4) and (9) permit one to determine the aerosol scattering optical depth τ_d with acceptable accuracy from observations of the absolute brightness phase functions $f(\varphi)$. The limits of applicability of the method are $0.09 \le \tau_d \le 0.4$; $2 \le \sec Z_0 \le 5$; and $0.4 \le \lambda \le 0.65$ μm.

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