CALCULATION OF OPTICAL TRANSFER FUNCTIONS OF THE ATMOSPHERE

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Numerical data on the atmospheric optical transfer functions, obtained by the spherical harmonics, Monte-Carlo, source function, and iteration methods are compared. The calculations were carried out using atmospheric models.

INTRODUCTION

Optical transfer functions of the atmosphere, determined calculationally by the solution of boundary-value problems of radiation transfer theory, are used in simulations of the brightness fields of outgoing short-wave radiation and to correct the remote sensing data on the underlying surface for atmospheric perturbations. The methods of calculation of these functions are discussed in a great number of papers, see, e.g., Refs. 1–9, but the absence of a common set of tested calculational algorithms causes many difficulties for users, connected with the search for and utilization of reliable algorithms and programs. In this connection, a check of the accuracy and computational speed of various calculation algorithms based on matched comparative tests remains very urgent.

Comparisons of some calculational algorithms of radiation transfer characteristics in plane layers, simulating the atmosphere, clouds, and mists, have been carried out in Refs. 2–4. Reference 8 presents the results of a comparison of values of the optical transfer functions of the cloudless three-dimensional atmosphere, obtained using the spherical harmonics, Monte-Carlo, and source function methods. In Ref. 9 similar calculations are carried out by the iteration method. In Refs. 8 and 9 optical models of the atmosphere were used.[10,11] In the present paper a more general comparison of optical transfer functions is performed based on the use of numerical data from Refs. 8 and 9. The main object of this paper is to draw the attention of the developers of these programs to the testing of the numerical methods of radiation transfer theory applied to optical models of the atmosphere. A comparison of the different calculational programs is needed in order to classify them with respect to accuracy and computational speed.

OPTICAL MODELS

Two optical models of the atmosphere were used. The first model (model I) was taken from Ref. 9 for the wavelengths 0.55 and 0.75 μm. The second one (model II) was taken from Ref. 10 for the wavelengths 0.3471, 0.6943, and 1.06 μm. The distinguishing feature of model I is that the atmospheric layer (h = 50 km) is divided into 50 sublayers, the values of $\alpha(z)$ and $\sigma(z)$ are specified at the sublayers' boundaries, the scattering phase function $f(\cos\gamma)$ is constant with respect to height, the values $f(\cos\gamma)$ are given with a 5° angular increment, and with a 1° angular increment in the range in which it varies strongly; at the points γ = 1,2,3,4° the values $f(\cos\gamma)$ were obtained by interpolation[7] of the tabulated data.[9] In the second model the atmospheric layer (h = 30 km) is divided into 35 sublayers; the values of $\alpha(z)$ and $\sigma(z)$ are also specified at the sublayer boundaries; with regard for the law of light scattering the atmosphere is considered to be three-layered, the values $f^{(i)}(\cos\gamma)$, I = 1, 2, 3, are assigned on a denser grid than in model I; the values of the scattering phase function in the forward direction are given in 2° angular increments because the functions $f^{(i)}(\cos\gamma)$ in this model are less elongated than in model I; the molecular scattering $f_R(\cos\gamma) = \dfrac{3}{16\pi}(1+\cos^2\gamma)$ was taken into account only at λ = 0.3471 μm, the total scattering phase function being calculated using the formula $f^{(i)}(\cos\gamma) = c_a^{(i)} f_a^{(i)}(\cos\gamma) + c_R^{(i)} f_R(\cos\gamma)$, i = 1, 2, 3, where $c_a^{(i)} = \tau_a^{(i)} / \tau_0^{(i)}$; $c_R^{(i)} = \tau_R^{(i)} / \tau_0^{(i)}$; and $\tau_0^{(i)} = \tau_a^{(i)} / \tau_R^{(i)}$, where $\tau_a^{(i)}$ and $\tau_R^{(i)}$ are the aerosol and Rayleigh optical depths in the i-th layer.

The radiation transfer model was determined by the boundary value problem in the three-dimensional non-spherical atmosphere, bounded by a surface with nonuniform albedo:

$$\begin{cases} (s, \vec{\nabla} l) + \alpha(z) I = \dfrac{\sigma(z)}{4\pi} \displaystyle\int_{\Omega} I(z, \vec{r}, \vec{s}\,') f(\vec{z}, \cos\gamma) d\vec{s}\,'; \\ I \Big|_{z=0,\, s\in\Omega_+} = \pi S_\lambda \delta(\vec{s} - \vec{s}_0); \\ I \Big|_{z=h,\, s\in\Omega_-} = \dfrac{q(\vec{r})}{\pi} \displaystyle\int_{\Omega_+} I(h, \vec{r}, \vec{s}\,') \mu' d\vec{s}\,', \end{cases} \tag{1}$$

where $I \equiv I(z, \vec{r}, \vec{s}) \equiv I_\lambda(z, x, y, \theta, \varphi)$ is the spectral radiation brightness; $\vec{r} = \{x, y\}$ is the vector of horizontal coordinates; $\vec{s} = \{\mu, \vec{s}_\perp\}$ is a unit vector; $\mu = \cos\theta$; $\vec{s}_\perp = \sqrt{1-\mu^2}\{\cos\varphi, \sin\varphi\}$; θ and φ are the zenith and azimuth angles- of the direction of propagation of the radiation; $\vec{s}_0 = \{\zeta, \sqrt{1-\zeta^2}, 0\}$ is the direction of incidence of the rays from the Sun; $\zeta = \cos\theta_0$; θ_0 is the zenith angle of the Sun; Ω_- and Ω_+ are the upper and lower hemispheres; πS_λ is the solar constant, W/(μm · cm^2 · st); $q(\vec{r})$ is the surface albedo; $\alpha(z)$ is the extinction coefficient; $\sigma(z)$ is the three-dimensional scattering coefficient; z is the vertical coordinate; h is the geometrical depth of the atmosphere; $f(z, \cos\gamma)$ is the scattering phase function; and $\cos\gamma = \vec{s}\cdot\vec{s}'$, where $\vec{s}$ and $\vec{s}'$ are the directions of the scattered and incident rays, respectively.

The solution of the boundary value problem (1) to within a nonlinear component relative to the variation $\tilde{q}(\vec{r})$ for the directions $\vec{s} \in \Omega_-$ has the form[6]

$$I = D + \frac{\bar{q}E\Psi_0}{1-\bar{q}c_0} + \frac{E}{1-\bar{q}c_0}\cdot\frac{1}{(2\pi)^2}\cdot$$
$$\cdot\int_{-\infty}^{\infty}\frac{\Psi(\vec{z}, \vec{p}, \vec{s})\hat{\tilde{q}}(\vec{p})e^{-i(\vec{p},\vec{r})}}{1-\bar{q}C(p)}d\vec{p}, \qquad (2)$$

where $D \equiv D(z, \mu, \zeta, \varphi)$ is the brightness of the atmospheric haze; $\pi E = \pi\left[\frac{1}{\pi}\int_0^{2\pi}\int_0^1 D(h, \mu, \zeta, \varphi)\mu d\mu d\varphi + \right.$ $\left. + \zeta S_\lambda e^{-\tau_0/\zeta}\right]$ is the illuminance of the Earth's surface at $\bar{q} = 0$ averaged over the horizontal coordinates; $\Psi(z, \vec{p}, \vec{s}) = e^{i(\vec{p},\vec{r})}[e^{-\tau_0/\eta} + Ae^{i\phi}]$ is the optical spatial-frequency characteristic of the atmosphere; $\Psi_0 = e^{-\tau_0/\eta} + A_0$; $c_0 = 2\int_0^1 \Psi_0(h, \mu)\mu d\mu$ is the spherical albedo; $\eta = |\mu| = \cos\Theta'$; $\Theta' = \pi - \Theta$; $A \equiv A(z, \vec{p}, \vec{s})$ and $\Phi \equiv \Phi(z, \vec{p}, \vec{s})$ are the amplitude and phase characteristics of the atmosphere an image spatial frequency filter; $\vec{p} = \{p_x, p_y\}$ is the spatial frequency vector; $A_0 \equiv A_0(z, \mu) = A(z, \vec{p}, \vec{s})|_{\vec{p}=0}$ is the norm of the atmospheric amplitude characteristic; $\bar{q}$ and $\tilde{q}(\vec{p})$ are the average value and the Fourier spectrum of the variation of the albedo of the underlying surface; $C \equiv C(\vec{p}) = \frac{1}{\pi}\int_{\Omega_+}\Psi(h, \vec{p}, \vec{s})\mu d\vec{s}$, and $\vec{r} = \vec{s}_\perp h/\eta$ is the displacement vector.

It is natural to use the optical transfer functions D, E, A_0, c_0, A, Φ, and C, which determine the action of the atmospheric transfer operator, as objects for test calculations. Note that if A and Φ are known, the point spread function $v(\vec{r}) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} Ae^{i\phi - i(\vec{r},\vec{p})}d\vec{p}$ evaluated in the general case by the fast-Fourier transform method.

NUMERICAL METHODS AND CALCULATIONAL RESULTS

As is well known from the literature, the functions D, E, A_0, c_0, A, Φ, and C are usually evaluated using the iteration[5,9] and spherical harmonics methods.[9,12,13] The Monte-Carlo method was used only for calculating the function A (Ref. 1) in addition to the $\vec{p}$-independent quantities D, E, A_0, and c_0. The functions D, E, A_0, c_0, and A were also calculated using an approximation technique.[6,16]

The errors of the Monte-Carlo method (MCM) and the iteration (IM) method depend on the number of simulated photon paths and on the difference scheme used. The errors of the spherical harmonics (SHM) and source function (SFM) methods depend on the order of the P_{2N+1} – approximation. The number N is generally determined at the stage at which the scattering phase function is expanded into a series over the Legendre polynomials. The actual calculational error can be higher than the error of the method because it is influenced by certain standard calculational procedures. In this work the MC[1,17], SH[12,13] and SF methods based on the P_1 – approximation (SFM_1)[6,16] were used. The latter method is the most approximate and is useful only because of the simplicity of its realization in systems of operational information processing.

In the comparisons below we will assume the MCM and IM calculations as the reference ones.[9] In the MCM algorithm the photon path simulation is made by estimating its direction. It is assumed that in the first two collisions the photon neither escapes from the medium nor is absorbed, while the corresponding displacement is taken into account by weighting factors. Direct simulation of the path is introduced beginning with the third collision. The variance of the MCM error in the calculation of D, E, A_0, c_0 was equal on average to 1%. The relative IM error in the calculations was also equal on average to approximately 1%. The accuracy of the other algorithms was estimated for all intents and purposes by comparing numerical results. Table I shows which optical characteristics can be obtained using each of the corresponding programs.

TABLE I.

Possibilities of the calculational programs.

Method	Optical transfer functions					
	D	E	A_0	c_0	A	Φ
SHM	+	+	+	+	+	+
SFM_1	+	+	+	+	+	–
MCM	+	+	+	+	–	–

The functions A and Φ were not calculated using MCM. The calculations of the function Φ by SFMˆ are characterized by poor accuracy and do not yield physically reasonable dependences $\Phi = \Phi(\vec{p})$.

In all of the algorithms the intensity of single scattering is evaluated analytically. In the SHM and MCM a piecewise-constant approximation of the coefficients $\alpha(z)$ and $\sigma(z)$ was used. In the SFM_1 realization true absorption was not taken into account. In the calculations of the function A the altitude dependence $\sigma(z)$ was approximated by an exponential, and the scattering phase function of the three-layer atmosphere by the function $f(\cos\gamma) = \sum_{i=1}^{3} \tau_0^{(i)} f^{(i)}(\cos\gamma) / \tau_0$. The time required for the computations of one variant, including the calculation of E and c_0, as well as the angular dependences of D and A_0 for two values of φ at fixed θ_0, using the MCM, SHM, and SFM_1 programs on an ES-1045 computer, was 10 min, 2 min, and 30 sec, respectively.

The transfer function calculations for the above-mentioned optical models of the atmosphere were carried out up to the factor of S_λ. The values of the input parameters were taken to be $z = 0$, $\varphi = 0$ and 180°, $\theta = 0$ and 45°. Figure 1 shows the angular dependences of D and A_0 for the cases $\lambda = 0.75$ μm (model I) and l$\lambda = 0.3471$ μm (model II). Table II displays the corresponding values of E and c_0.

Figures 2 and 3 show the normalized amplitude A/A_0 and phase Φ characteristics calculated for $\lambda = 0.3471$ μm. For comparison in Figs. 1–3 and in Table II the results of the IM calculations are given as well.[9]

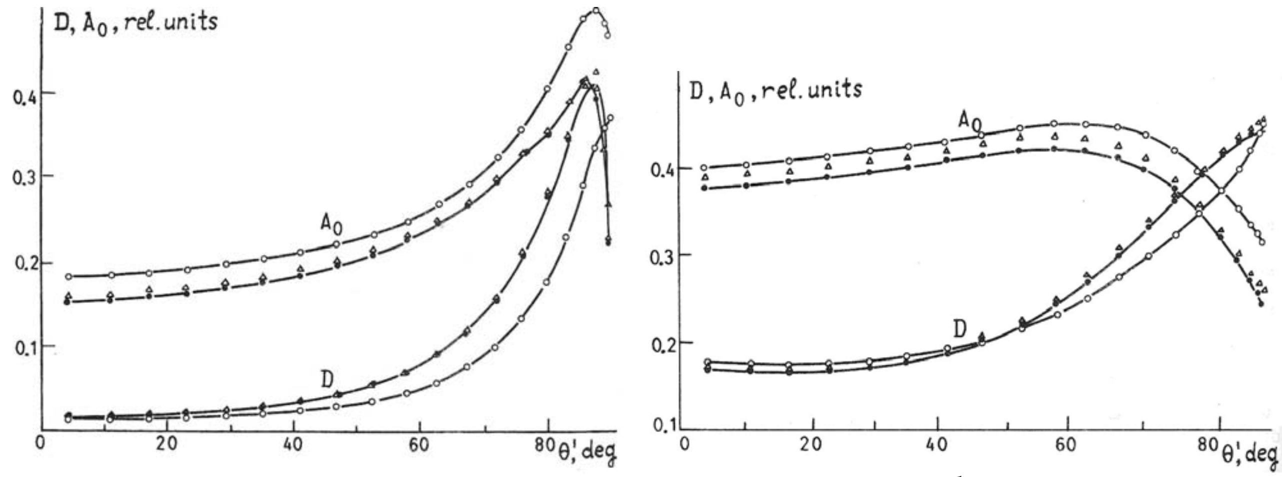

FIG. 1. The angular dependences of D and A_0 by model I (a) and model II (b) for $\theta_0 = 45°$, $\varphi = 0$; dots – SHM; circles – SFM_1; triangles – IM.

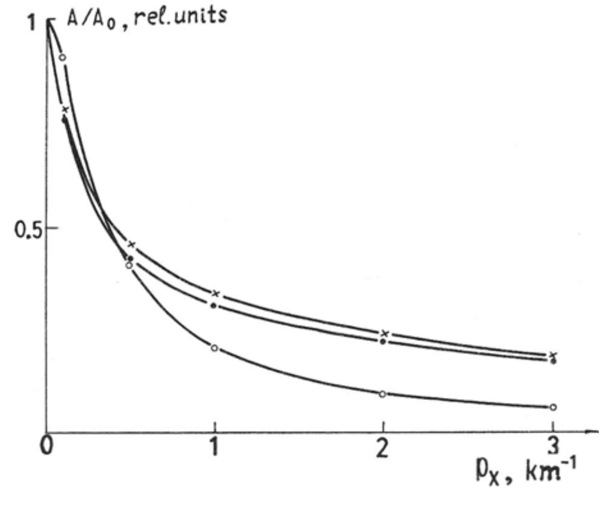

FIG. 2. Normalized amplitude-frequency characteristic A/A_0 according to model II for $\eta = 0.997$, $\varphi = 0°$, $P_y = 0$: dots – SHM; circles – SFM_1; crosses – IM.

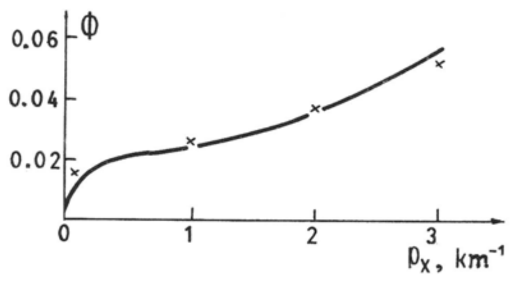

FIG. 3. Phase characteristic by model II for $\eta = 0.997$, $\varphi = 180°$, $P_x = 0$: solid line – SHM; crosses – IM.

TABLE II.

Values of optical transfer functions obtained by different methods.

Method	λ=0.75 μm		λ=0.3471 μm	
	E	c_0	E	c_0
IM	0.6530	0.1036	0.4406	0.3598
SHM	0.6491	0.1049	0.4348	0.3563
SFM_1	0.6670	0.0865	0.4518	0.3839

The calculations carried out showed that the values of D, E, A_0, and c_0 obtained by SHM and MCM practically coincide. That is why the calculated values of these quantities, obtained by MCM, are not shown in Fig. 1 or Table II. As one can see from Figs. 1–3, the SHM data agree well with the IM data.[9] When the results of the calculations using SHM (dots) and IM (triangles) coincided, the latter were not marked in the plot. Despite the good agreement between the SHM calculations and the IM calculations of the functions Φ, it is necessary to note the following fact. In Fig. 3 the function $\Phi = \Phi(p_x)$ has a nonmonotonic derivative. If the natural assumption about the monotonic behavior of $d\Phi/dp_x$ is made, then it becomes necessary to conclude that the errors of the given calculations of Φ using SHM and IM exceed 1%. The same conclusion follows from the results of numerical experiments, which show that Φ is more sensitive to calculational errors than A.

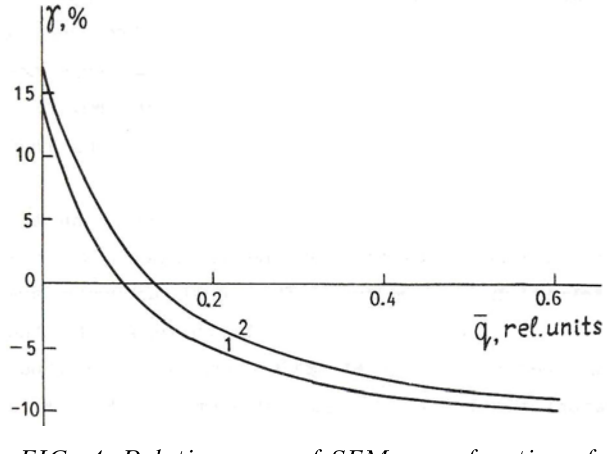

FIG. 4. Relative error of SFM_1 as a function of the average albedo of the underlying surface according to model I for $\lambda = 0.55$ μm, $\theta_0 = 45°$, $\varphi = 0$: 1 – $\eta = 0.997$; 2 – $\eta = 0.818$.

The error of $SFM_1\hat{}$ depends noticeably on the optical characteristics of the atmosphere and the observational geometry. The least accurate are the calculations of the brightness of the atmospheric haze D at $\theta > 30°$ and the amplitude characteristic A at $|p| > 0.5$ km^{-1}. Figure 3 shows the relative error of the calculation of the average brightness of the outgoing radiation ($\gamma = [(\bar{I}_{SHM} - \bar{I}_{SFM_1})/\bar{I}_{SHM})] \cdot 100\%$) as a function of average albedo of the underlying surface. It is easy to see that the SFM_1 error for the observation angles $\theta' \leq 30°$ and albedo $\vec{q} \geq 0.05$ in the cases shown in Fig. 4 does not exceed 10%. A similiar estimation is obtained for the other λ values as well.

CONCLUSION

This paper presents results of tests of the calculational algorithms for solving the problem of radiation transfer in the atmosphere. Calculations of the optical transfer functions of the atmosphere by the Monte-Carlo, spherical harmonics, and source function methods, based on the P_1 approximation, showed a qualitative coincidence of the results. A comparison .of the calculational results showed that in the calculation of the functions D, E, A_0, c_0, A, and Φ the spherical harmonics method[12,13] is just as good in accuracy as the Monte-Carlo[17] and iteration[9] methods, and is substantially faster. The SFM_1 approximation is useful for calculating the brightness of the radiation field with an error ≤ 10%, provided that $\theta' \leq 30°$ and $\vec{q} \geq 0.05$

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